International Journal of Innovative Research in Advanced Engineering (IJIRAE) ISSN: 2349-2163
Issue 2, Volume 2 (February 2015) www.ijirae.com

An Application of Fractional Calculus in RLC Circuit

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Abstract:-- In this paper, an attempt as an application, we obtain the solution of fractional differential equation associated with a RLC electrical circuit, using Heaviside function in a closed form in terms of the Mittag-Leffler function.

Mathematics Subject Classification: 26A33, 33E12

Keywords: RLC Electrical circuit, Mittag-Leffler function and its extensions, Fractional derivatives.

1. INTRODUCTION:

The application of the Mittag-Leffler function and its extension are discussed [17] recently in a rapidly increasing numbers of papers, related to fractional calculus and fractional order differential and integral equations and systems modling in various phenomena.

2. DEFINITION:

Parabolic function:
Let $f(t)$ be the parabolic function. The ramp mathematically expressed as follows:

$$f(t) = \begin{cases} 0 & for\; t < 0 \\ k\frac{t^2}{2} & for\; t \geq 0 \end{cases} \tag{1}$$

And its Laplace transform is$\frac{1}{s^3}$.

3. *RLC* ELECTRICAL CIRCUIT

In this paper, we present RLC electrical circuit with a capacitor and an inductor are connected in parallel and this set is connected in series with a resistor and voltage. The capacitance C, the inductance L and the resistor R are consider positive constants and $\theta(t)$ is the ramp function earlier paper [17], consider the $\theta(t)$ is Heaviside function.

The constitutive equations associated with a three elements of *RLC* electrical circuit are:
The voltage drop

$$U_L(t) = L\frac{d}{dx}I(T), \qquad \text{across an inducter;}$$

The voltage drop

$$U_R(t) = RI(t), \qquad \text{across a resistor;}$$

The voltage drop

$$U_c(t) = \frac{1}{C}\int_0^t I(\xi)d\xi, \qquad \text{across a capacitor}$$

And where $I(t)$ is the current.
Applying the Kirchhoff's voltage law and constitutive equations associated with the three elements, we can write the non-homogeneous second order ordinary differential equation

$$L\frac{d^2}{dt^2}U_C(t) + R\frac{d}{dt}U_C(t) + \frac{1}{L}U_C(t) = \frac{d}{dt}\theta(t) \tag{2}$$

Where $U_c(t)$ is the voltage on the capacitor, this is the same on the inductor as we can see in figure 1, because they are connected in parallel. On the other hand, we obtain other non-homogeneous second order ordinary differential equations associated with the current on the capaciter,

$$\frac{L}{C}\frac{d}{dt}i_C(t)+\frac{R}{C}i_C(t)+\frac{1}{C}\int_0^t i_C(\xi)d\xi,=\frac{d}{dt}\theta(t) \tag{3}$$

We note that, integro-differential equations have the some form. Here we consider only the first one. The classical methodology to discuss this integro-differential equation is the Laplace transform. To this end, we consider the initial condition $i_C(0)=0$ and the solution can be found in terms of an exponential function [24].

4. FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION

In this section we discuss the fractional form of equation (3), i.e. a Fractional integro-differential equation associated with a current on the capacitor,

$$\frac{L}{C}\frac{d^\alpha}{dt^\alpha}i_C(t)+\frac{R}{C}i_C(t)+\frac{1}{C}\frac{1}{\Gamma\alpha}\int_0^t (t-\xi)^{\alpha-1}i_C(\xi)d\xi,=\frac{d}{dt}\theta(t) \tag{7}$$

With $0<\alpha\leq 1$, and the fractional derivative is taken in the Caputo sense, where $\theta(t)$ is the Heaviside function. In this case, one can thought that $i_C(t)$ can be interpreted as a Green's function because the second member is a delta function. We also consider $i_C(0)=0$, i.e., the initial current on the capacitor is zero. We note that this equation is a possible generalization of the classical integro-differential equation associated with the RLC electrical circuit, because for $\alpha=1$ we recover the results obtained in Subsection 3. This replacement can be useful in discussing the corresponding numerical problem, for a particular value of the parameter, because the solution is presented in terms of a closed expression.
To solve this fractional integro-differential equation, we introduce the Laplace integral transform, defined by

$$L[i_C(t)]=F(s)=\int_0^\infty e^{-st}i_C(t)\,dt \tag{4}$$

with $Re(s)>0$, and we obtain the following algebraic equation

$$\frac{L}{C}s^\alpha F(s)+\frac{R}{C}F(s)+\frac{1}{C}\frac{F(s)}{s^\alpha}=1, \tag{5}$$

Whose solution is given by

$$F(s)=\frac{C}{L}\frac{s^\alpha}{s^{2\alpha}+as^\alpha+b}, \tag{6}$$

where we have introduced the positive parameters $a=R/L$ and $b=1/L$.

To recover the solution of the fractional integro-differential equation, we proceed with the inverse Laplace transform

$$i_C(t)=\frac{C}{L}L^{-1}\left[\frac{s^\alpha}{s^{2\alpha}+as^\alpha+b}\right] \tag{7}$$

Using the relation [3]

$$L^{-1}\left[\frac{s^{\rho-1}}{s^\alpha+As^\beta+B}\right]=t^{\alpha-\rho}\sum_{r=0}^{\infty}(-A)^r t^{(\alpha-\beta)r}E^{r+1}_{\alpha,\alpha+1-\rho+(\alpha-\beta)r}(-Bt^\alpha)$$

$$\text{Valid for }\left|\frac{As^\beta}{s^\alpha+B}\right|<1\text{ and }\alpha\geq\beta,$$

We can write,

$$i_C(t)=\frac{C}{L}t^{\alpha-1}\sum_{r=0}^{\infty}(-a)^r t^{\alpha r}E^{r+1}_{2\alpha,\alpha+\alpha r}(-bt^{2\alpha})\,\theta(t) \tag{8}$$

Where $E^{\rho}_{\mu,v}(t)$is the three parameter Mittag-Leffler functions and $\theta(t)$ is the Heaviside function.

Again, if we consider $\theta(t)$ function is Parabolic function then the solution

$$i_C(t)=\frac{C}{L}t^{\alpha+1}\sum_{r=0}^{\infty}(-a)^r t^{\alpha r}E^{r+1}_{2\alpha,\alpha+2+\alpha r}(-bt^{2\alpha})\,\theta(t) \tag{9}$$

CONCLUSION:

In this paper we obtain new results for series in three-parameter Mittag- Leffer functions. The possible applications of our results, we obtain a closed form to the solution of the fractional integro- differential equation associated with a particular *RLC* electrical circuit, in terms of the three-parameter Mittag-Leffer function.
Our main result is interesting with respect to simplifying several other results, for i.e. as one can see in [5] where we discussed the fractional telegraph equation, and in [4], where the anomalous diffusion was presented. The results in both papers are given in terms of the three- parameter Mittag-Leffer function.

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