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Control Analysis of a mass- loaded String

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***Abstract*— This study deals with the active control of the dynamic response of a string with fixed ends and mass loaded by a point mass. It has been controlled actively by means of a feed forward control method. A point mass of a string is considered as a vibrating receiver which be forced to vibrate by a vibrating source being positioned on the string. By analyzing the motion of a string, the equation of motion for a string was derived by using a method of variation of parameters. To define the optimal conditions of a controller, the cost function, which denotes the dynamic response at the point mass of a string was evaluated numerically. The possibility of reduction of a dynamic response was found to depend on the location of a control force, the magnitude of a point mass and a forcing frequency.**

***Keywords*— Active control, a point mass, feed forward control method, a method of variation of parameters, cost function**

I. Introduction

In the industrial fields, vibration phenomena of machines and structures had been serious problems to be solved for the green environmental conditions. So far a lot of researches have been performed to analyze and control the vibration phenomena induced by machining operations in the past decades. Currently the fast manufacturing time and the accurate level of the machining tools are known to be the important factors of the optimal design condition. In case of the needs of the high speed operation, the vibration phenomenon should be one of the overcoming troubles for the stability of the machine structure.

For the simplification of the complex structures, each part of the machines is considered as a continuous system such as string, beam, plate or shell. Hence, the vibration characteristics of a continuous system had been studied by lots of persons. Recently for the investigation of the vibration characteristics, the vibration energy flow and the dynamic response of a beam, plate, shell or some compound system have been analyzed. By using a model of an elastic beam, the vibration energy and control technologies had been presented [1-3]. Furthermore, the complex frame which is the assembly of several beams had been used to analyze and control the vibration characteristics [4-5]. The feedforward control method had been proven to become the convenient control method for the known forcing frequency zone [6-8].

In this study a vibrating system which consists of a point mass and a string with a primary source and a control force is modelled to analyze and control the dynamic response of a string. The edges conditions of a string are fixed. Based on the wave equation, the vibration of a string will be discussed. To define the optimal conditions for the control of the dynamic response, the cost function will be evaluated numerically by using the Mathmatica program of 'FindMinimum'. Fig. 1 shows the theoretical model of a uniformly stretched string which is mass-loaded by a point mass and excited to vibration in the plane.

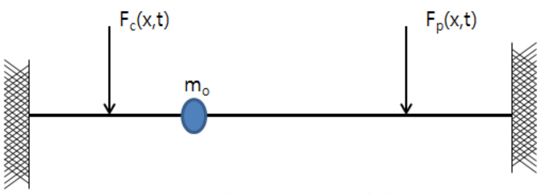

Fig. 1: Theoretical model
F_p (source), F_c (control force), A point mass (mass (m_o)),
String properties (length (L), density (ρ), constant tension (T))

II. The Governing Equation

A uniformly stretched string of mass density ρ and length L fixed at both ends is subjected to a point mass m_o located at a distance of x_o from the left end of the string. The governing equation of a vibrating mass-loaded string can be written as:

$$T\frac{\partial^2 u(x,t)}{\partial x^2} = \rho\frac{\partial^2 u(x,t)}{\partial t^2} + m_o\delta(x-x_o)\frac{\partial^2 u(x,t)}{\partial t^2} + F_p\delta(x-x_p)e^{i\omega t} + F_c\delta(x-x_c)e^{i\omega t}. \quad (1)$$

Where *u(x,t)* represent the displacement of a string and ω is the circular frequency. In Eq. (1), the source and control forces are given as the time harmonic forcing function. The displacement (*u(x,t))* is expressed into the terms of the time harmonic motion as,

$$u(x,t) = y(x)e^{i\omega t}. \quad (2)$$

Where *y(x)* means the deflection of a string. By inserting Eq. (2) into Eq. (1) and then suppressing the time term, Eq.(1) can be rearranged as,

$$\frac{\partial^2 y(x)}{\partial x^2} + k^2\, y(x) = -\frac{m_o\omega^2}{T} y(x_o)\delta(x-x_o) + \frac{F_p}{T}\delta(x-x_p) + \frac{F_c}{T}\delta(x-x_c). \quad (3)$$

Where the wave number $k^2=\rho\omega^2/T$. The total solution of Eq. (3) can be expressed into the sum of a homogeneous solution (y_h) and a particular solution (y_p) as

$$y(x) = y_h(x) + y_p(x). \quad (4)$$

The homogeneous solution is determined by letting the right side of Eq. (3) be zero and then becomes as

$$y_h(x) = A\sin kx + B\cos kx. \quad (5)$$

Where the constants A and B can be obtained by using the boundary conditions of a string. Here the particular solution can be solved by means of a method of variation of parameters and then be assumed as

$$y_p(x) = V_1(x)\sin kx + V_2(x)\cos kx. \quad (6)$$

Where the coefficients V_1 and V_2 are can be determined by means of the method of variation of parameters and defined as follows

$$V_1(x) = \frac{1}{k}\int^x \cos(k\xi) f(\xi)d\xi \qquad \textit{and} \qquad V_2(x) = -\frac{1}{k}\int^x \sin(k\xi) f(\xi)d\xi.$$

$$\textit{where the forcing function}, \quad f(\xi) = -\frac{m_o\omega^2}{T} y(x_o)\delta(\xi - x_o) + \frac{F_p}{T}\delta(\xi - x_p) + \frac{F_c}{T}\delta(\xi - x_c).$$

The final form of the particular solution can be obtained by using Eqs. (5) and (6). The complete solution of Eq. (3) can be written as

$$y(x) = A\sin kx + B\cos kx - \frac{F_p}{kT}\sin(k(x-x_p))H(x-x_p) - \frac{F_c}{kT}\sin(k(x-x_c))H(x-x_c)$$
$$-\frac{m_o\omega^2}{kT} y(x_o)\sin(k(x-x_o))H(x-x_o). \quad (7)$$

Where H(x) represents the Heaviside unit step function and the quantity, *y(x_o)* can be evaluated by inserting $x=x_o$ into Eq. (7) and then becomes as

$$y(x_o) = A\sin kx_o + B\cos kx_o - \frac{F_p}{kT}\sin(k(x_o-x_p))H(x_o-x_p) - \frac{F_c}{kT}\sin(k(x_o-x_c))H(x_o-x_c).$$

By inserting $y(x_o)$ into Eq. (7) and then using the boundary conditions, the final solution of the dynamic response of a string becomes as,

$$y(x) = \frac{A1}{A2}\left[\sin kx - \frac{m_o\omega^2}{kT}\sin kx_o \sin k(x-x_o)H(x-x_o)\right]$$
$$-\frac{F_p}{kT}[\sin k(x-x_p)H(x-x_p) - \frac{m_o\omega^2}{kT}\sin k(x-x_o)H(x-x_o)\sin k(x_o-x_p)H(x_o-x_p)]$$
$$-\frac{F_c}{kT}[\sin k(x-x_c)H(x-x_c) - \frac{m_o\omega^2}{kT}\sin k(x-x_o)H(x-x_o)\sin k(x_o-x_c)H(x_o-x_c)]. \quad (8)$$

Where the constant functions A1 and A2 are as follows,

$$A1 = -\frac{F_p}{kT}[\sin k(L-x_p) - \frac{m_o\omega^2}{kT}\sin k(L-x_o)\sin k(x_o-x_p)H(x_o-x_p)]$$
$$-\frac{F_c}{kT}[\sin k(L-x_c) - \frac{m_o\omega^2}{kT}\sin k(L-x_o)\sin k(x_o-x_c)H(x_o-x_c)]$$

and

$$A2 = \sin kL - \frac{m_o\omega^2}{kT}\sin kx_o \sin k(L-x_o).$$

Equation (8) is employed as the control factor to be reduced actively by applying the control force to a string. So the cost function for the reduction of dynamic response of a point mass can be defined as

$$\Pi = [\mathrm{Real}|y(x_o)|]^2. \quad (9)$$

Here the optimal value of the control force which leads to the minimum value of Eq. (9) can be obtained numerically as the following process,

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$$\nabla_f \Pi \Rightarrow Minimization\ of\ \Pi\left(f_c \rightarrow f_{optimal}\right) \tag{10}$$

Where the control force is evaluated in terms of the nondimensional form which be based on the source force. As mentioned before, the optimal values have been obtained by using Mathmatica program -'FindMinimum'.

III. NUMERICAL RESULTS AND DISCUSSION

All values obtained in this study have been expressed into the nondimensional forms. In Table 1, the nondimensional properties and the given values are introduced as

TABLE I
NONDIMENSIONAL PROPERTIES AND THE GIVEN VALUES

	Expression form	Value
A mass ratio	$m = m_o/(m_o+\rho L)$	variable
Source force	Fp	1
A structural damping ratio		0.001
A point mass location	$Xo=x_o/L$	0.4
Source location	$Xp=x_p/L$	0.6
Control force location	$Xc=x_c/L$	variable
Control force	fc=Fc/Fp	

A. Resonance Frequency Equation

In Eq. (8), the denominator part A2 vanishes for certain specific values of the wave number k. The roots of denominator which are expressed in ω_r by $k^2=\rho\omega^2/T$ are called the resonance frequencies for the string. So the resonance frequency equation becomes as,

$$F(k) = \sin kL - \frac{m_o\omega^2}{kT}\sin kx_o \sin k\left(L - x_o\right) = 0. \tag{11}$$

Here Eq. (11) can be rewritten in terms of the nondimensional form as,

$$F(\alpha) = \sin \alpha\mu - m\frac{\alpha}{\mu}\sin \alpha\mu X_o \sin \alpha\mu\left(1 - X_o\right) = 0. \tag{12}$$

$$Where\quad a\ nondimensional\ frequency\ (\alpha) = \frac{\omega}{\sqrt{T/L(m_o+\rho L)}}\quad and\quad \mu = \sqrt{1-m}$$

Equation (12) is a transcendental equation in α and its roots must be obtained numerically. The first two roots of Eq. (12) were found numerically. In Fig. 1, the first (α_1) and the second (α_2) resonance frequency loci for four different mass ratios (m=0, 0.1, 0.2, 0.3) are plotted versus the location of the point mass (X_o). As the magnitude of a point mass is smaller, the less variance be experienced. For the first resonance frequency, as the location of a point mass is close to the center, the values are getting smaller in comparison with no point mass case. But in case of the second resonance frequency, the gap between the maximum and minimum values is getting bigger than that of the first resonance frequency.

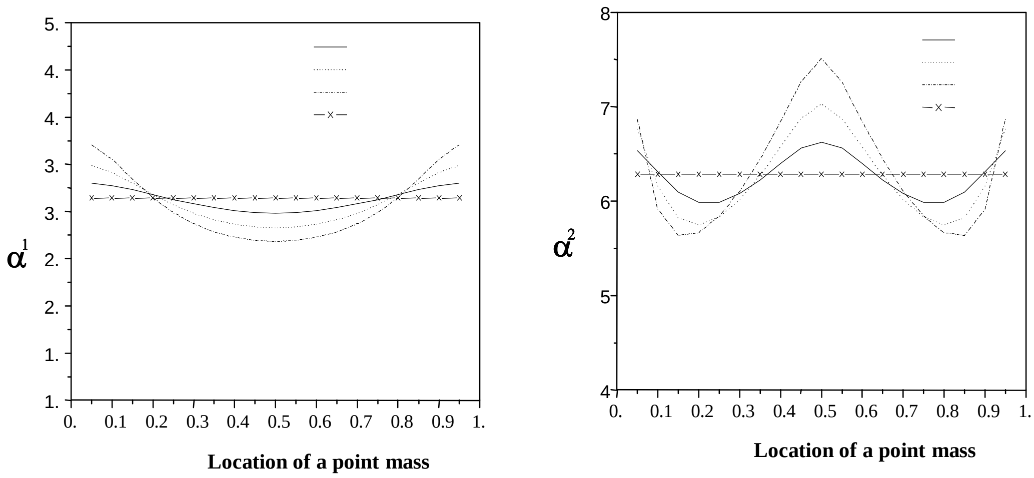
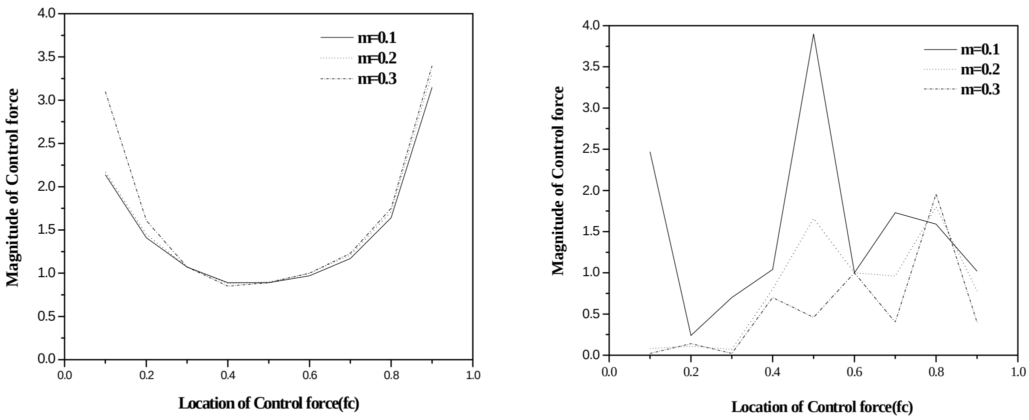


Fig. 2 A nondimensional frequency vs. location of a point mass : α_1 and α_2

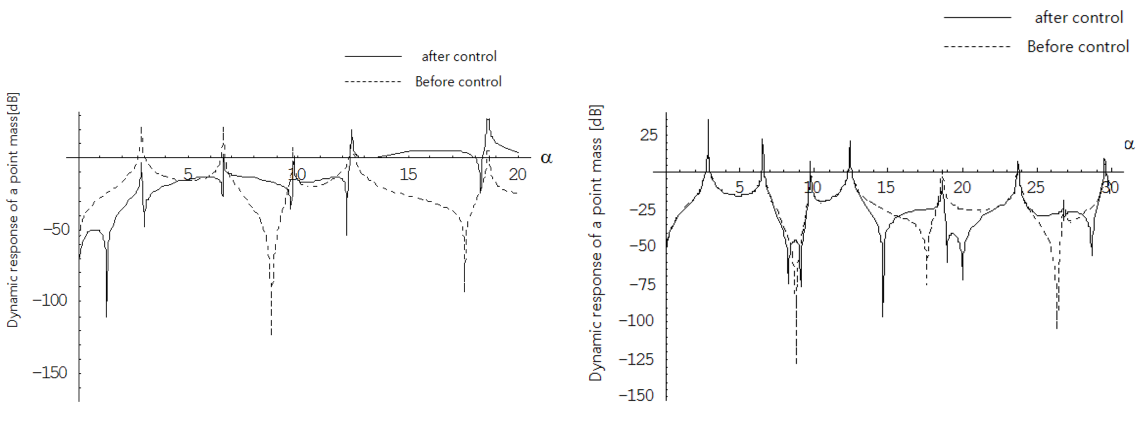
B. Reduction of Dynamic Response of a Point mass

The reduction level of the dynamic response of a point mass is expressed in the value of decibel [dB] as $20Log_{10}[y(x_o)]$. The optimal control force was evaluated by Eq. (10) with the fixed locations which are the source location (X_p=0.6) and the point mass location (X_o=0.4). In Fig. 3, (A) and (B) show the variations of control force versus its location for two cases of nondimensional frequencies (α=3, 20), respectively. For the low frequency zone (α=3), the regular pattern of force loci were found for three different mass ratios. As the control force is located closely to the center of string, the magnitude is getting smaller. However at high frequency zone (α=20), the magnitudes of control force are too changable to be estimated. In Fig. 4, the dynamic responses of a point mass were plotted against nondimensional frequency (α). In case of Fig. 4 (A), the dynamic response of a point mass was controlled only for α=3. Hence, the reduction was done well for the low frequency zone (α=1~5). But for the other frequency zone, the given optimal control force is getting ineffective or worse to reduction level. Similar case is shown in Fig. 4 (B) for α=20.



(A) (B)

Fig. 3 Magnitude of Control force vs. location of Control force for three mass ratio : Xp=0.6, Xo=0.4, (A) α=3 and (B) α=20



(A) (B)

Fig. 4 Dynamic response of a point mass vs. α: Xp=0.6, Xo=0.4, (A) α=3 and (B) α=20

IV. Conclusions

On the bases of the analyses in this paper, the conclusions are obtained as follows, - Feedforward control method is proven to give the satisfactory results for the reduction of the dynamic response of a point mass which is forced to vibrate by the source of the string. In Fig. (5), the reduction values in dB were plotted versus location of Control force for three nondimensional frequencies (α=3, 9, 20). All values of reduction are known to be over 300 dB. It is surely noted that the reduction of dynamic response of a point mass be done successfully up to zero deflection level.

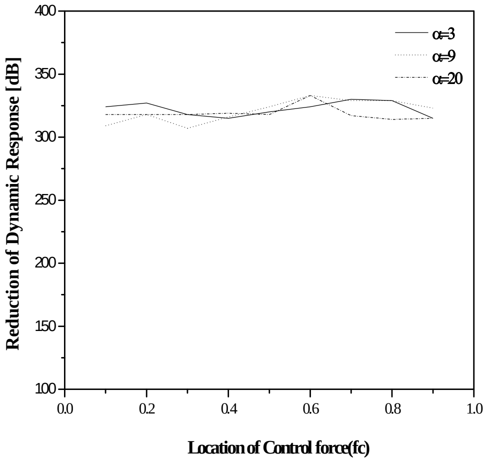


Fig. 5 Reduction of Dynamic response [dB] vs. Location of Control force: Xp=0.6, Xo=0.4, m=0.2

- The location of the control force is known to become the important factor for the control strategy.
- At the low frequencies, the optimal locations of the control force are known to be positioned close to the center of the string.

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