FEM Analysis of Natural Convection Flows in Porous Trapezoidal Enclosure

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Abstract — Simulations were carried out for natural convection in a trapezoidal cavity filled with porous media to investigate the effect of uniformly and non-uniformly varying heat flux on bottom wall using finite element computational procedure. The enclosure used for flow and heat transfer analysis has bottom wall subjected to heat flux, constant temperature cold top wall and adiabatic side walls. The bottom wall is subjected to uniform / linear /sinusoidally varying heat fluxes. Results are presented in the form of stream lines, isotherms, heatline plots and average Nusselt numbers. Nusselt numbers are computed for Darcy-modified Rayleigh numbers or Rayleigh number (Ra) ranging from 100 to 1000 for an aspect ratio (H/L) of 0.5 of cavity. It is observed from this study that the average Nusselt number is more for sinusoidally varied heat flux of the bottom wall. The power law correlations between average Nusselt number and Rayleigh numbers a represented for convection dominated regions.

Keywords: Natural convection, Porous medium, Galerkin’s finite element method, Trapezoidal cavity, Heat flux

I. INTRODUCTION

Natural convection heat transfer in a porous trapezoidal cavity with bottom wall subjected to heat flux is investigated. Flow in the porous enclosure has a considerable importance in various practical situations like in buildings in which heat is transferred across an insulation filled enclosure [1]. A two dimensional trapezoidal cavity having one horizontal wall with heat flux boundary condition considered is as shown in figure 1. The inclined walls are assumed to be insulated. In most practical situations, constant heat flux boundary conditions may be more suitable than constant temperature conditions. Finite element method [2] is a numerical technique for finding approximate solutions to boundary value problems.

II. NUMERICAL ANALYSIS

Henry Darcy derived various relations of the momentum equation which is the porous-medium analog to Navier-stokes equation. The variation of density with respect to temperature is given by Boussinesq approximation. The continuity equation can be automatically satisfied by introducing stream function $\psi$ as: $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Fig. 1 Trapezoidal cavity
Continuity and Energy equations are the two governing partial differential equations in dimensional form with many variables. These equations can be non-dimensionalised which reduces the number of variables and thus facilitates the solution. The following non-dimensional parameters are used to convert above said equations into non-dimensional form:

\[
\theta = \frac{T - T_0}{qL_{ref}/k}
\]

Temperature

\[
\psi = \frac{\Psi}{\alpha}
\]

Stream function

\[
Ra = \frac{gL_4k}{\mu\alpha}
\]

Rayleigh number

Substitution of above non-dimensional terms gives rise to following non-dimensional equations:

\[
\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = Ra \frac{\partial \theta}{\partial X} \tag{1}
\]

\[
\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} - \frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} - \frac{\partial}{\partial X}(U\theta) - \frac{\partial}{\partial Y}(V\theta) \tag{2}
\]

\[
\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} - \frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} - \frac{\partial}{\partial X}(U\theta) - \frac{\partial}{\partial Y}(V\theta) \tag{3}
\]

Eq. (1) and (2) are two coupled partial differential equations as change of a variable in one equation affects the other equation. H is heat function related to non-dimensional velocity and temperature as given in equation 3. Galerkin’s method is employed to convert the partial differential equation into matrix form of equation for an element. The present study is carried out by using simple 3 noded triangular elements. Prasad [3] studied natural convection in a rectangular porous cavity with constant heat flux on one vertical wall. Natural convection in a porous annulus with constant heat flux is studied by Prasad [4]. Peter Vadasz [5] derived analytical solutions to confirm experimental and numerical results revealing a widespread dispersion of heat flux data in natural convection in porous media. Maria Neagu [6] studied analytically heat and mass transfer induced by a constant heat and mass fluxes wavy wall in a non-Darcy double stratified porous medium.

**III. RESULTS & DISCUSSIONS**

The cavity used for the analysis is subjected to uniform heat flux at the bottom wall. Computations are carried out for Rayleigh number ranging from 100 to 1000. The aspect ratio is considered as 0.5. The cavity is heated with uniform heat flux from the bottom wall and cooled from the top wall. q is applied heat flux and k is the thermal conductivity of the porous medium. The average Nusselt number is obtained as:

\[
Nu = \frac{1}{\int (\theta_h - \theta_c) dl}
\]

In this study the constant, linearly varying and sinusoidally varying heat flux has been investigated at the bottom wall. The same amount of heat input is maintained in all three cases.

### 3.1 Uniform Heat Flux at the Bottom Wall

Flow fields are represented by streamlines with equal increment ΔΨ, temperature distributions are represented by isotherms with equal increment ΔT and heat energy distributions are represented by heatlines with equal increment ΔH. As Ra is increased, there is slight increase in the magnitude of stream function values. From the isothermal patterns, it is clear that the maximum non-dimensional temperature decreases with an increase in Rayleigh number as shown in Figure 2. Although the dimensional temperature will increase with Rayleigh number, the dimensionless temperature decreases due to temperature scale selected with heat flux in the denominator. Distribution of heat energy is represented by heatlines. The shapes of streamlines and heatlines are found to be almost elliptical. In the heatline pattern, cells are seen in which heat is rotated.
3.2 Linear Heat Flux Variation at the Bottom Wall

The heat flux at bottom wall is varying linearly and same amount of heat flux is given as in the earlier case. In contrast, for linearly varying heat flux variation, profiles are not symmetric about central vertical line. For a value of $Ra=1000$, multiple circulation cells are formed as observed in figure 3 and dimensionless temperature $\theta$ decreases. However, for case of heat flow energy, only positive value of heatlines drawn with an increase of magnitude towards right.

Fig. 2 Streamlines, Isotherms and Heatlines for bottom wall subjected to uniform heat flux boundary condition for $Ra=1000$. 
Fig. 3 Streamlines, Isotherms and Heatlines for bottom wall subjected to linearly varying heat flux boundary condition for $Ra=1000$.

3.3 SINUSOIDAL HEATFLUX VARIATION AT THE BOTTOM WALL

Figure 4 show contours for $Ra=1000$ when the bottom wall is subjected to sinusoidal varying heat flux boundary condition. It is noticed that streamlines are of single cell. Inner loop is of circular shape lesser $Ra$ value of 100 and elliptical with convection dominant Rayleigh numbers. Elliptical loop spreads over the cavity for $Ra=1000$. Isotherms tend towards right of the cavity with lesser value of dimensionless temperature as the value of Rayleigh number increases.
Fig. 4 Streamlines, Isotherms and Heatlines for bottom wall subjected to sinusoidally varying heat flux boundary condition for $Ra = 1000$

Figs. 5 and 6 shows the variation of average Nusselt number versus Rayleigh number for bottom wall and top wall respectively for all the three cases. It can be observed that the average Nusselt number increases with Rayleigh number as expected. The average Nusselt number for the top wall as in Fig 6 is less as compared to the bottom hot wall, as expected.

![Fig. 5 Variation of Average Nusselt number of bottom wall](image)

![Fig. 6 Variation of Average Nusselt number of top wall](image)

Correlations have been drawn between Average Nusselt number and Rayleigh number as shown in following table.

<table>
<thead>
<tr>
<th>HEATING TYPE</th>
<th>RAYLEIGH NUMBER RANGE</th>
<th>BOTTOM WALL</th>
<th>CORRELATION</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform heat flux</td>
<td>100 to 1000</td>
<td>$\bar{N}u = 0.141Ra^{0.496}$</td>
<td>0.997</td>
<td></td>
</tr>
<tr>
<td>Linear varying heat flux</td>
<td>100 to 1000</td>
<td>$\bar{N}u = 0.291Ra^{0.436}$</td>
<td>0.988</td>
<td></td>
</tr>
<tr>
<td>Sinusoidal varying heat flux</td>
<td>100 to 1000</td>
<td>$\bar{N}u = 0.26Ra^{0.495}$</td>
<td>0.988</td>
<td></td>
</tr>
</tbody>
</table>
TABLE II
CORRELATION OF NUSSELT NUMBER WITH RAYLEIGH NUMBER FOR TOP WALL

<table>
<thead>
<tr>
<th>HEATING TYPE</th>
<th>RAYLEIGH NUMBER RANGE</th>
<th>TOP WALL CORRELATION</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform heat flux</td>
<td>100 to 1000</td>
<td>( \overline{Nu} = 0.916Ra^{0.018} )</td>
<td>0.984</td>
</tr>
<tr>
<td>Linear varying heat flux</td>
<td>100 to 1000</td>
<td>( \overline{Nu} = 0.956Ra^{0.01} )</td>
<td>0.984</td>
</tr>
<tr>
<td>Sinusoidal varying heat flux</td>
<td>100 to 1000</td>
<td>( \overline{Nu} = 0.609Ra^{0.007} )</td>
<td>0.992</td>
</tr>
</tbody>
</table>

IV. CONCLUSIONS

In the present study, the investigations have been carried for the bottom wall subjected to different heat flux boundary conditions like constant, linearly varied and sinusoidally varied heat flux cases (total heat supplied at the bottom wall is same in each case). The top wall is cold and side walls are maintained adiabatic. The following points have been observed during the present study.

- The contours of stream functions and isotherms are symmetric about vertical line at centre for constant but they are not symmetric for linearly varying and sinusoidally varying heat flux cases.
- The average Nusselt number increases monotonically with increase of Ra
- The average Nusselt number is more for sinusoidally varied heat flux for bottom wall.

REFERENCES

[6]. Maria Neagu (2011) Free convective heat and mass transfer induced by a constant heat and mass fluxes vertical wavy wall in a non Darcy double stratified porous medium. IJHMT, 54, 2310-2318.