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Cubic BF-Algebra

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Abstract— Y.B.Jun et al. [9] introduced the notion of Cubic sets and Cubic subgroups. In this paper we introduced the notion of cubic BF- Algebra i.e., an interval-valued BF-Algebra and an anti fuzzy BF-Algebra. Intersection of two cubic BF- Algebras is again a cubic BF-Algebra is also studied.

Keywords—BF-Algebra, Fuzzy BF-sub algebra, interval-valued Fuzzy sets, interval-valued BF-sub algebra, Cubic BF-sub algebra.

I. Introduction

After L.A.Zadeh's [15] introduction of interval-valued (i-v) fuzzy sets, where the values of the membership functions are interval of real numbers instead of the real points, there was much important in this field. In 1966, Y.Imai and K.Iseki [5] introduced the two classes of abstract algebras: BCK-algebras and BCI-algebras. Andrzej Walendziak [1] introduced a more generalized a class of algebra named BF- Algebra. A.Borumand Saeid and M.A.Rezvani [2] introduced the concept of fuzzy BF-Algebras. A.Zarandi and A.Borumand Saeid [16] introduced the notion of i-v fuzzy BF-Algerbras. Recently, Y.B.Jun et al. [10] introduced the notion of cubic sets and cubic subgroups. Moreover, Y.B.Jun et al.[9] studied the concept of Cubic subalgebras and ideals of BCK/BCI-algebras.

This paper is an attempt to introduced the notion of Cubic BF-algebra and provide results on it.

II. PRELIMINARIES

In the following we provide the essential definitions and results necessary for the development of our theory.

Definition 2.1[1]. A BF-algebra is a non-empty set X with a constant 0 and a binary operation * satisfying the following axioms:

(i) $x * x = 0$,
(ii) $x * 0 = x$,
(iii) $0*(x*y) = y*x$ for all $x,y \in X$.

Example 2.2[1]. Let R be the set of real numbers and A=(R; *, 0) be the algebra with operation * defined by

$$x * y = \begin{cases} x, if\ y = 0 \\ y, if\ x = 0 \\ 0.\ otherwise \end{cases}$$

Then A is a BF-Algebra.

Example.2.3 [1]. Let $A = [0,\infty)$. Define the binary operation * on A as follows:

$x*y = |x - y|$, for all $x, y \in A$. Then (A; ∗, 0) is a BF-Algebra.

Definition.2.4[1]. A non-empty subset S of a BF-algebra X is called a subalgebra of X if $x* y \in S$, for any $x,y \in S$.

A mapping $f: X\to Y$ of BF-algebra is called BF –homorphism if $f(x*y) = f(x)*f(y)$, for any $x, y\in X$.

We now review some fuzzy logic concepts (see[12]).Let X be a set .A fuzzy set A in X is characterized by a membership function $\mu_A: X \to [0,1]$.Let f be a mapping from the set X to the set Y and let B be a fuzzy set in Y with membership function μ_B. The inverse image of B, denoted $f^{-1}(B)$,is the fuzzy set in X with membership function $\mu_{f^{-1}(B)}$ defined by $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ for all $x\in X$. Conversely, let A be a fuzzy set in X with membership function μ_A Then the image of A, denoted by f(A),is the fuzzy set in Y such that:

$$\mu_{f(A)}(y) = \begin{cases} \sup \mu_{f(A)}(z), if\ f^{-1}(y) = \{x: f(x) = y\} \neq \varphi \\ 0,\ otherwise \end{cases}$$

An interval valued fuzzy set (briefly, i-v fuzzy set) A defined on X is given by

$$A=\{(x,[\mu_A^L(x),\mu_A^U(x)])\}, \forall x \in X$$

Briefly, it is denoted by $A = [\mu_A^L, \mu_A^U]$ where μ_A^L and μ_A^U are any two fuzzy sets in X such that $\mu_A^L(x) \leq \mu_A^U(x)$ for all $x \in X$.

Let $\overline{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$, for all $x \in X$ and let D[0,1] denote the family of all closed sub – intervals of [0,1]. It is clear that if $\mu_A^L(x) = \mu_A^U(x) = c$, where $0 \leq c \leq 1$ then $\overline{\mu}_A(x) = [c, c]$ is in D[0,1]. Thus $\overline{\mu}_A(x) \in D[0,1]$, for all $x \in X$. Therefore, an i-v fuzzy set A is given by

$A = \{(x, \overline{\mu}_A(x))\}$, for all $x \in X$ where $\overline{\mu}_A : X \to D[0,1]$.

Now we define the refined minimum (briefly, rmin) and order "≤"on elementary $D_1=[a_1,b_1]$ and $D_2 = [a_2, b_2]$ of D [0,1] as:

$$\text{rmin}(D_1, D_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}],$$
$$D_1 \leq D_2 \Leftrightarrow a_1 \leq a_2 \wedge b_1 \leq b_2.$$

Similarly we can define ≥ and =.

Definition 2.5 [2]. Let μ be a fuzzy set in a BF – algebra X. Then μ is called a fuzzy BF- sub algebra (BF- Sub algebra) of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 2.6 [15]. Let X be a non-empty set. A cubic set A in a set X is a structure

$A = \{< x, \overline{\mu}_{A(x)}, \lambda_{(x)} > : x \in X\}$ which is briefly denoted by $A = < \overline{\mu}_A, \lambda >$, where $\overline{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval valued fuzzy set (briefly, IVF) in X and $\lambda : X \to [0,1]$ is a fuzzy set in X. Denote by C (x) the family of cubic sets in a set X.

Definition 2.7[9]. A cubic set $A = < \overline{\mu}_A, \lambda >$ in X is called a cubic subgroup of X if it satisfies for all x, y ∈ X,

(a) $\overline{\mu}_A(xy) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$

(b) $\overline{\mu}_A(x^{-1}) \geq \overline{\mu}_A(x)$

(c) $\lambda(xy) \leq \max\{\lambda(x), \lambda(y)\}$

(d) $\lambda(x^{-1}) \leq \lambda(x)$

Example 2.8[9]. Let X be the Klein's four group. We have X ={e, a ,b, ab}, where $a^2 = e = b^2$ and ab = ba. We define $\overline{\mu}_A = [\mu_A^-, \mu_A^+]$ and λ by

$$\overline{\mu_A} = \begin{pmatrix} e & a & b & ab \\ [0.5,0.8] & [0.4,0.6] & [0.1,0.5] & [0.1,0.5] \end{pmatrix}$$

and $$\lambda = \begin{pmatrix} e & a & b & ab \\ 0.2 & 0.3 & 0.6 & 0.5 \end{pmatrix}$$

Then $A = < \overline{\mu}_A, \lambda>$ is a cubic subgroup of X.

Definition 2.9[16]. An i-v fuzzy set A in x is called an interval-valued fuzzy BF-subalgebra (briefly i-v fuzzy BF -Algebra) of X if

$\bar{\mu}_A(x*y) \geq \text{rmin}\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ for all x, y ϵ X.

Definition 2.10. Let A and B be two fuzzy subsets of X. Then the cartesian product A and B is defined by $(A \times B)(x, y) = \min\{A(x), B(y)\}$, for all x, y in X.

III. CUBIC BF -ALGEBRA

We now introduced the notion of cubic BF-Algebra.

Definition 3.1. Let λ be a fuzzy set in a BF- algebra X. Then λ is called an anti fuzzy BF - sub algebra (BF-Algebra) of X if $\lambda(x*y) \leq \max\{\lambda(x), \lambda(y)\}$ for all x , y ϵ X.

Definition 3.2. A cubic set $A = <\bar{\mu}_A, \lambda>$ in X is called a BF- subalgebra(BF-Algebra) of X if it satisfies for all x, y ϵ X.

(a) $\bar{\mu}_A(x*y) \geq \min[\bar{\mu}_A(x), \bar{\mu}_A(y)\}$

(b) $\lambda(x*y) \leq \max\{\lambda(x), \lambda(y)\}$

Example 3.3. Let X = { 0, 1, 2, 3} be a set with the following table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

Then (x, *, 0) is a BF- Algebra.

Define

$$\bar{\mu}_A(x) = \begin{cases} [0.3, 0.9], if\ x \in \{0,2\} \\ [0.1, 0.6], otherwise \end{cases}$$

and

$$\lambda(x) = \begin{cases} 0.9, if\ x \in \{1,2,3\} \\ 0,\ if\ x = 0 \end{cases}$$

It is easy to check that $A = <\bar{\mu}_A, \lambda>$ is cubic BF – Algebra.

Lemma 3.4. If $A = <\bar{\mu}_A, \lambda>$ is a cubic BF -Algebra of X then for all x ϵ X

(i) $\bar{\mu}_A(0) \geq \bar{\mu}_A(x)$

(ii) $\lambda(0) \leq \lambda(x)$

Proof. For all x ϵ X, we have

(i) $\bar{\mu}_A(0) = \mu_A(x*x)$

$\geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(x)\} = \bar{\mu}_A(x) \Rightarrow \bar{\mu}_A(0) \geq \bar{\mu}_A(x)$

(ii) $\lambda(0) = \lambda(x * x)$
$\leq \max\{\lambda(x), \lambda(x)\} = \lambda(x) \Rightarrow \lambda(0) \leq \lambda(x)$

Theorem 3.5. The intersection of two cubic BF-Algebras is again a cubic BF -Algebra.

Proof. Let x , y ϵ $A_1 \cap A_2$. Then x, y ϵA_1 and A_2, since A_1 and A_2 are i-v fuzzy BF-Algebra.

$$\bar{\mu}_{A1\cap A2}(x*y) = \min\{\bar{\mu}_{A1}(x*y), \bar{\mu}_{A2}(x*y)\}$$
$$\geq \min\{\min[\bar{\mu}_{A1}(x), \bar{\mu}_{A1}(y)], \min[\bar{\mu}_{A2}(x), \bar{\mu}_{A2}(y)]\}$$
$$\bar{\mu}_{A1\cap A2}(x*y) \geq \min\{\bar{\mu}_{A1\cap A2}(x), \bar{\mu}_{A1\cap A2}(y)\}$$

Let λ and γ be two anti fuzzy BF-algebra of X. Since x,y ϵ $\lambda \cap \gamma$ then x ,y ϵ λ and x,y ϵ γ.

We have $(\lambda \cap \gamma)(x*y) = \min\{\lambda(x*y), \gamma(x*y)\}$
$$\leq \min\{\max[\lambda(x), \lambda(y)], \max(\gamma(x), \gamma(y)\}$$
$$\leq \max(\min[\lambda(x), \lambda(y)], \min(\gamma(x), \gamma(y)\}$$
$$= \max\{(\lambda \cap \gamma)(x), (\lambda \cap \gamma)(y)]$$

$\Rightarrow (\lambda \cap \gamma)(x*y) \leq \max\{(\lambda \cap \gamma)(x), (\lambda \cap \gamma)(y)\}$, which proves the Theorem.

Corollary 3.6. A family of a cubic BF-Algebra is again a cubic BF-Algebra.

Theorem 3.7. The Cartesian product of two cubic BF-Algebras of X is a cubic BF-Algebra of X.

Proof. (i) Let $\bar{\mu}_A$ and $\bar{\mu}_B$ are two i-v fuzzy BF-Algebras of X.

Define $(x_1, y_1) * (x_2, y_2) = (x_1 * x_2, y_1 * y_2)$ for all x_1, x_2, y_1, y_2 in X.

Then $(\bar{\mu}_A \times \bar{\mu}_B)((x_1, y_1) * (x_2, y_2)) = (\bar{\mu}_A \times \bar{\mu}_B)(x_1 * x_2, y_1 * y_2)$
$$= \min\{\bar{\mu}_A(x_1 * x_2), \bar{\mu}_B(y_1 * y_2)\}$$
$$\geq \min\{\min[\bar{\mu}_A(x_1), \bar{\mu}_A(x_2)], \min[\bar{\mu}_B(y_1), \bar{\mu}_B(y_2)]\}$$
$$\geq \min\{\min[\bar{\mu}_A(x_1), \bar{\mu}_B(y_1)], \min[\bar{\mu}_A(x_2), \bar{\mu}_B(y_2)]\}$$
$$= \min\{(\bar{\mu}_A \times \bar{\mu}_B)(x_1, y_1), (\bar{\mu}_A \times \bar{\mu}_B)(x_2, y_2)\}$$

(ii) Let λ and γ be the two anti fuzzy BF-Algebras.

Now, $(\lambda \times \gamma)((x_1, y_1) * (x_2, y_2)) = (\lambda \times \gamma)(x_1 * x_2, y_1 * y_2)$
$$= \max\{\lambda(x_1 * x_2), \gamma(y_1 * y_2)\}$$
$$\leq \max\{\max[\lambda(x_1), \lambda(x_2)], \max[\gamma(y_1), \gamma(y_2)]\}$$
$$\leq \max\{\max[\lambda(x_1), \gamma(y_1)], \max[\lambda(x_2), \gamma(y_2)]\}$$
$$= \max\{(\lambda \times \gamma)(x_1, y_1), (\lambda \times \gamma)(x_2, y_2)\}$$

Hence, the Cartesian product of cubic BF-Algebra of X is a cubic BF-Algebra of X.

IV. CONCLUSIONS

In this paper, we have extended the notion of the bipolar fuzzy BF-Algebra of X and α-cut of the bipolar fuzzy BF-Algebra of X. Moreover, we discuss the concept of cubic topological BF-Algebra.

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