

Short-Cycle Reduction Algorithm in Parity-Check Matrix of an Irregular LDPC code to Improve Error Floor Rate and Computational Complexity

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Abstract— Short-length low-density parity-check (LDPC) codes have many interesting application in wireless communications. Sparse parity-check matrix 'H' is the basis for the LDPC code formation. Short cycles of length 4 in a matrix 'H' of short-length LDPC codes are cause error. Hence, removable of such short cycles is major challenge in code construction process. In this paper, a code construction algorithm, namely reduction of short cycles (RSCA) is proposed, which aims to minimize the short-cycles length 4, without searching these short cycles directly in the parity-check matrices. H matrix consists of sub matrix p^T and identity matrix I_{n-k} . One of the randomly generated p^T sub matrices is selected on the basis of fitness value. Best fit sub matrix is appended with I_{n-k} to form a final parity-check matrix H. The designed matrix H contains optimal number of short-cycles of length 4. Simulation results show that the proposed algorithm significantly reduces the short cycles and improves performance the in terms of BER, FER, computational complexity (CC).

Keywords— LDPC codes, fitness value, sparse parity-check matrix, AWGNC, BPSK modulation

I. INTRODUCTION

LDPC codes have shown excellent error correcting capabilities approaching the Shannon limit [1,5]. These codes shall be represented by sparse parity-check matrix 'H' that contains minimum non-zero entries or by a graph known as Tanner graph (TG). TG, consists of variable nodes (dv) corresponds to columns and check nodes (dc) corresponds to rows of sparse parity-matrix [6]. If the number of non-zero elements of 'H' matrix is fixed in column and rows, codes are called regular LDPC codes else an irregular LDPC codes. Major difference between other linear block error correcting codes and LDPC codes is that LDPC codes requires sparse parity-check matrix 'H' and it should not possess a short cycle of length 4 (fig.1.a). It is the sparseness of 'H' which guaranteed a low decoding complexity and a minimum distance. LDPC codes are designed with construction of a sparse parity-check matrix 'H'. Generator matrix 'G' is determined from 'H' to encode the k-bit messages. TG is formed by connecting variable nodes dv and check nodes dc. The degree of variable and check nodes depends on number of non-zero elements in the column & rows respectively. If sparse parity-check matrix is represented in systematic form as, $H = [P^T I_{n-k}]$, then the generator matrix $G = [I_k P]$, where, P is known as sub matrix. The sub matrix P is not sparse; hence, the encoding complexity is quite high. If the block length increases the encoding complexity go up by $O(n^2)$, even sparse matrix should not help in such condition and produce good result. Encoded bits are modulated, transmitted through AWGN channel and decoded using the sum-product algorithm (SPA) or message passing algorithm (MPA). Large length LDPC codes are better performing because their TGs contains less number of short cycles, but its implementation is not practical in many applications due construction complexity or more construction time [1,7]. Short-length codes are of many applications but their performance may degrade a lot if there exists a large number of short cycles of length 4. The LDPC codes have already been adopted in satellite-based digital video broadcasting, Long-haul optical communication standards, adopted in the IEEE wireless local area network standard, long-term evolution of third generation mobile telephony.

II. RELATED WORK

Short length LDPC codes with large girth have been proposed in [2-4],[6,13-14],[17]. The error floor issue of short-length LDPC codes have been addressed in [4,6-19]. In [10] error floor of short length LDPC codes over (BEC) is discussed. In [11] it is shown that small size stopping set degrades the performance of short-length LDPC codes over AWGN channel and code matrices with low error floor is obtained by avoiding short cycles with ACE value below a given threshold. Good performing LDPC codes at waterfall and high SNR region is studied in [11, 14]. In [8, 12] A Trapping sets (TSs) is introduced to evaluate the performance of LDPC codes at high SNR values. However, it is well known that the trapping set without short cycles do not contribute to the error floor. In [15]-[19] author shown minimization of error floor-rate and computational complexity in short length LDPC codes. Hence, the focus of entire work carried here was to formulate an algorithm which generates the optimistic parity-check matrices with optimum number short cycles by short cycle reduction method. Objective is to construct short-length irregular LDPC codes with low error floors. In this paper, a concept of fitness value (fv) is introduced to select the parity-check matrix H corresponding to best fit value of sub matrix p^T . An algorithm for code construction for minimization of short cycle in short-length LDPC codes is proposed with best fit methodology. The performance of Codes built in systematic form is studied in terms of parameter BER, FER and computational complexity (CC).

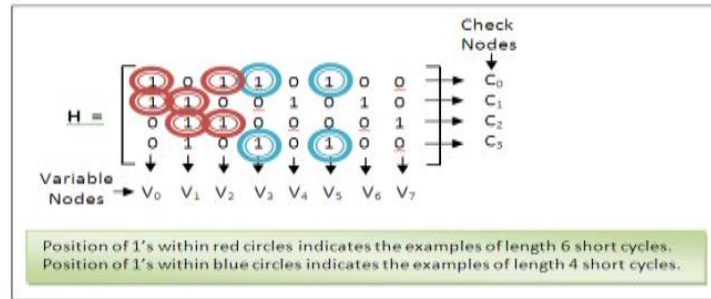


Fig.1 a. parity-check matrix H (n-k, n)

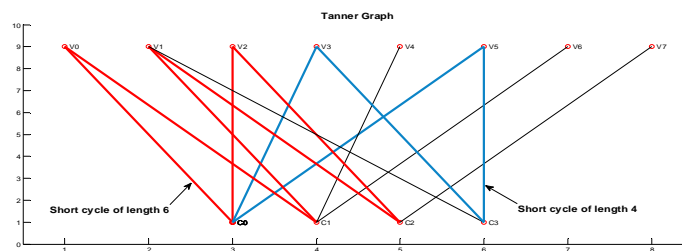


Fig.1.b. Tanner graph corresponding to H matrix

The organization of paper is as follows, in section II dealt with the related work, in section III proposed algorithm for code construction, in section IV simulation results, finally we conclude the paper in section V.

III. PROPOSED (SCMA) ALGORITHM

Methodology for best fit selection:

Evaluation of the code is done with following parameter;

$BER = N / 100 * f_bits$,

$FER = n_frames / 100$,

Where, n_frame is no. of erroneous frame,

N- Total no. of erroneous bits,

F_bits is no. of bits per frame

If the FER or BER is considered for the threshold it may not lead to optimization result. Hence, collectively the parameter is considered for selection of the best fit matrix as Fitness parameter which is reciprocal of sum of average value of BER and FER with various SNR values.

The fitness parameter is used for selection of best fit sparse matrix 'H' out of randomly generated ones.

SCMA

Begin;

1. Select the size of 'H' matrix, No. of column=n (code length), no. of rows =n-k
2. Initialize the sub matrices p_1^T, p_2^T, \dots
3. Evaluate each sub matrices p_1^T, p_2^T, \dots over AWGN channel and MPA decoder
4. Find best sub matrix out of p_1^T, p_2^T, \dots comparing their fitness value
5. Construct matrix $H = [p^T \ I_{n-k}]$ in this form
6. Construct generator matrix 'G' from 'H' matrix as $G = [I_k \ p]$
7. Encode the 100, k-bit messages to form n-bit codeword and decode received codeword using final 'H' matrix
8. The performance parameter such as BER, FER and CC are recorded evaluation purpose

Given an LDPC codes, we make the use of aforementioned method to reduce the short cycles in the sub matrices and hence in corresponding parity-check matrix H is formed by appending with identity matrix.

Table 1 Short cycles observed in sub matrices of parity-check matrices

Code length (n,k)	Short cycles length 4 in sub matrices and hence in H matrices				
	P_1^T	P_2^T	P_3^T	P_4^T	Reduction in short cycles
16,8	37	38	45	52	71%
16,32	773	784	798	807	95.8%
60,30	11123	11170	10332	11165	82.5%
64,32	13804	14050	14254	14366	76.8%
128,64	251143	245865	258179	250693	96%
256,128	4062705	4140004	4138461	4131666	74.9%

Table 2 BER vs. Eb/No (dB) (SNR) with systematic matrices

Code length (n,k),	Eb/N0 (dB)				
	0	1	2	3	4
(16,8)	0.0206	0	0	0	0
(32,16)	0.0359	0.0191	0	0	0
(60,30)	0.0808	0.0220	0.0037	0	0
(64,32)	0.0834	0.0233	0.0008	0	0

Table 3 FER vs. Eb/No (dB)(SNR) with systematic matrices

Code length n,k	Eb/N0 (dB)				
	0	1	2	3	4
16,8	0.0800	0	0	0	0
32,16	0.2100	0.1000	0	0	0
60,30	0.5100	0.1600	0.0500	0	0
64,32	0.5100	0.1900	0.0100	0	0

Table 4 CC with various code lengths for systematic matrices

Code length n,k	H matrices				
	8x16	16x32	30x60	32x64	64x128
CC(s)	0.2135	1.35	2.5771	3.3488	4.678

IV. RESULTS AND DISCUSSIONS

In the previous section, we have proposed a new short cycle minimization algorithm that finds optimal parity-check matrices from the randomly generated sub matrices i . Consider systematic matrices of size 8x16, 16x32, 32x64, 30x60, 64x128 and code rate 1/2. Four sub matrices in each case is generated, fitness value of each sub matrix is calculated. The sub matrix with highest fitness value is appended with I_{n-k} identity matrix to form final H matrix. This final designed H matrix is used to construct the LDPC code. Short cycles in each of the four sub matrices are counted. Observed values of short cycles in sub matrices for H matrices of size 8x16 are as follows- 37, 38, 45 and 52 respectively. Proposed approach obtains the best sub matrix p^T , which corresponds to short cycles of 37 which is minimum of all sub matrices $p_1^T, p_2^T \dots$ and hence corresponding H is constructed by appending with identity matrix of size (n-k) x (n-k) as $H = [p^T \ I_{n-k}]$.

ii. **Bit Error Rates (BER)**- Assuming AWGN channel, the sum-product algorithm or message passing algorithm will iterate a maximum of 50 times for each of received codeword's or till the correct code word is obtained. For each of codes corresponding to sub matrices and simulations will continue until 100 code words of given length are collected. Table 3 shows the values of BER vs. received signal-to-noise ratios (SNR) in dB. BER is obtained are 0, 0, 0, 0 with code length 16 bits, 32 bits, 60 bits and 64 bits respectively at SNR=4 dB.

iii. **Frame Error Rates (FER)**- For each of codes simulations will continue until 100 code words of given length are collected. Table 4 shows the values of FER vs. received signal-to-noise ratios (SNR) in dB or Eb/N0(dB). FER obtained are 0, 0, 0, 0 with code length 16bits, 32bits, 60 bits, 64bits respectively at SNR=4 dB

iv. Computational complexity (CC) / Construction time (CT)(seconds)-

Construction time or computational complexity for codes are tabulated and these are 0.2135, 1.35, 2.5771, 3.3488, 4.678 with code length 16, 32, 60 and 64, 128 respectively

V. CONCLUSIONS

In this paper, we have proposed a new short cycle minimization algorithm which uses a parameter called fitness value of sub matrices are computed to find the optimal parity-check matrix H in systematic form for irregular LDPC codes. Simulation results shows that code constructed with this algorithm achieves better reduction in short cycles and hence, improves performance in terms of BER, FER, CC

VI. REFERENCES

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