


Solutions of Interval Linear Systems Using Interval Gauss–Jacobi and Interval Gauss–Seidel Iterative Methods

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Abstract: This paper presents a study of iterative methods and interval arithmetic for solving systems of linear equations with interval coefficients. The Interval Gauss–Jacobi and Interval Gauss–Seidel methods are discussed for obtaining solutions to systems of interval linear equations. Numerical examples involving 2×2 and 3×3 interval systems are provided to demonstrate the implementation and effectiveness of these methods.

Key words: interval Arithmetic, Interval Linear Systems, Interval Gauss–Jacobi Method, Interval Gauss–Seidel Method, Iterative Methods, AMS Subject Calcifications: 65G40, 65F10, 65G30, 65N22, 15A06.

1. INTRODUCTION

This chapter introduces the concept of interval arithmetic, developed to manage uncertainties in numerical computations. This chapter forms the mathematical basis for solving systems with interval coefficients. Ramon E. Moore[4] is credited with formally introducing interval arithmetic in 1966. He systematized earlier ideas into a rigorous computational framework. His book Interval Analysis is considered the foundational reference of the field and helped establish interval arithmetic as a recognized area of numerical analysis. The main goal is to perform computations using intervals instead of exact numbers, so that results include guaranteed bounds on errors. This ensures that the true value of a computation is always contained within the computed interval.

1.1 Definition: An interval $[a, b]$ is the set of all real numbers between a and b where $a < b$. Let $X = [a, b]$ and $Y = [c, d]$ be two intervals.

(i). Addition of two intervals is defined as $[a, b] + [c, d] = [a + c, b + d]$

Example: If $A = [2, 5]$, $B = [1, 3]$, then $A+B = [2+1, 5+3] = [3, 8]$

(ii). Subtraction of two intervals is defined as $[a, b] - [c, d] = [a - d, b - c]$

Example. If $A = [4, 7]$, $B = [2, 6]$, then $A-B = [4-6, 7-2] = [-2, 5]$

(iii). Multiplication of two intervals is defined as

$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$

Example. If $A = [2, 4]$, $B = [3, 5]$, all the combinations are $2 \times 3 = 6$, $2 \times 5 = 10$, $4 \times 3 = 12$, $4 \times 5 = 20$, Then $A \times B = [\min(6, 10, 12, 20), \max(6, 10, 12, 20)] = [6, 20]$

(iv). Division of two intervals is defined as $X \div Y = X \times [1/d, 1/c]$, provided $0 \notin [c, d]$

Note. Division is undefined if 0 is inside the denominator interval.

Example. If $A = [6, 10]$, $B = [2, 5]$, First take reciprocal of B : $1/B = [1/5, 1/2]$

Now $A \div B = AX \ 1/B = [6, 10] \times [1/5, 1/2]$, all the combinations: $6 \times 1/5 = 6/5$, $6 \times 1/2 = 3$, $10 \times 1/5 = 2$, $10 \times 1/2 = 5$, then $A \div B = [6/5, 5]$

1.2. Definition: An interval matrix is a matrix in which each element is an interval rather than a single real number. If $A = (a_{ij})$ is a matrix, then in interval form, each element is written as $a_{ij} = [l_{ij}, u_{ij}]$, Where l_{ij} is the lower bound and u_{ij} is the upper bound and $l_{ij} \leq u_{ij}$

$$\text{Example for } 2 \times 2 \text{ interval matrix } A = \begin{bmatrix} [a_{11}, b_{11}] & [a_{12}, b_{12}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] \end{bmatrix}$$

$$\text{Example for } 3 \times 3 \text{ interval matrix } A = \begin{bmatrix} [a_{11}, b_{11}] & [a_{12}, b_{12}] & [a_{13}, b_{13}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & [a_{23}, b_{23}] \\ [a_{31}, b_{31}] & [a_{32}, b_{32}] & [a_{33}, b_{33}] \end{bmatrix}$$

2. INTERVAL GAUSS JACOBI ITERATION METHOD

In many practical problems, the coefficients and constants of a system of linear equations are not known exactly because of measurement errors, approximation, or uncertainty. Such systems are represented using interval numbers instead of fixed real numbers. To obtain approximate interval solutions for these systems, iterative numerical methods are applied. The Interval Jacobi Iteration Method is an extension of the classical Jacobi iterative method in interval arithmetic. In this method, every coefficient and unknown is represented as an interval, and successive approximations are obtained by repeatedly substituting previous interval values into the system equations. The Interval Jacobi Iteration Method is particularly useful for diagonally dominant interval systems because such systems generally ensure convergence. The method provides approximate interval solutions that contain the possible exact solutions of the uncertain system.

2.1. Interval Jacobi iteration method for system of n- linear equations

Consider a system of n-linear interval equations $AX=B$ where A is an interval coefficient matrix, X is the vector of unknown interval variables, and B is the interval constant vector. The method first rearranges each equation so that one variable is expressed in terms of the remaining variables. Starting from suitable initial interval approximations, new interval values are computed iteratively. The Jacobi method uses only the values from the previous iteration to compute the next iteration. This process continues until the difference between two successive iterations becomes negligibly small, indicating convergence of the interval solution.

Consider the system of n-linear equations

$$\begin{aligned} [a_{11}, b_{11}] x_1 + [a_{12}, b_{12}] x_2 + \dots + [a_{1n}, b_{1n}] x_n &= [c_1, d_1] \\ [a_{21}, b_{21}] x_1 + [a_{22}, b_{22}] x_2 + \dots + [a_{2n}, b_{2n}] x_n &= [c_2, d_2] \\ &\vdots \\ [a_{n1}, b_{n1}] x_1 + [a_{n2}, b_{n2}] x_2 + \dots + [a_{nn}, b_{nn}] x_n &= [c_n, d_n] \end{aligned}$$

From above equation of system (1), we have

$$\begin{aligned} x_1 &= \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2 + [a_{13}, b_{13}] x_3 + \dots + [a_{1n}, b_{1n}] x_n] \\ x_2 &= \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1 + [a_{23}, b_{23}] x_3 + \dots + [a_{2n}, b_{2n}] x_n] \\ &\vdots \\ x_n &= \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1 + [a_{n2}, b_{n2}] x_2 \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}] \end{aligned}$$

Let $x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}$ are interval

$$\begin{aligned} x_1^{(1)} &= \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2^{(0)} + [a_{13}, b_{13}] x_3^{(0)} + \dots + [a_{1n}, b_{1n}] x_n^{(0)}] \\ x_2^{(1)} &= \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1^{(0)} + [a_{23}, b_{23}] x_3^{(0)} + \dots + [a_{2n}, b_{2n}] x_n^{(0)}] \\ &\vdots \\ x_n^{(1)} &= \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1^{(0)} + [a_{n2}, b_{n2}] x_2^{(0)} \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}^{(0)}] \\ x_1^{(k+1)} &= \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2^{(k)} + [a_{13}, b_{13}] x_3^{(k)} + \dots + [a_{1n}, b_{1n}] x_n^{(k)}] \\ x_2^{(k+1)} &= \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1^{(k)} + [a_{23}, b_{23}] x_3^{(k)} + \dots + [a_{2n}, b_{2n}] x_n^{(k)}] \\ &\vdots \\ x_n^{(k+1)} &= \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1^{(k)} + [a_{n2}, b_{n2}] x_2^{(k)} \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}^{(k)}] \end{aligned}$$

For, $k=0,1, 2, \dots$. Continue the above process until we get the desired accuracy of the root. The following problem includes detailed step-by-step solutions for 2×2 interval systems and 3×3 interval systems.

2.2. Application to Interval Jacobi iteration method (Two interval linear equations)

This section illustrates the application of the Interval Jacobi Iteration Method to a 2x2 system of interval linear equations.

$$[7, 9]x + [0, 1] y = [8, 10] \text{ ——— (1)}$$

$$[1, 3]x + [6, 8] y = [12, 14] \text{ ——— (2)}$$

$$[7,9] > [0,1], [6,8] > [1, 3]$$

From equations (1) and (2), write iterative form:

$$\text{Eq(1)} \Rightarrow x = \frac{([8,10] - [0,1]y)}{[7,9]}, \quad \text{Eq (2)} \Rightarrow y = \frac{([12,14] - [1,3]x)}{[6,8]}$$

Consider the Initial values: $x^{(0)} = [0, 0]$, $y^{(0)} = [0, 0]$

The Iteration formula for Jacobi method is

$$x^{(k+1)} = \frac{([8,10] - [0,1]y^{(k)})}{[7,9]}, \quad y^{(k+1)} = \frac{([12,14] - [1,3]x^{(k)})}{[6,8]}$$

In this method we use previous iteration values only.

1st Iteration:

For $x^{(1)}$:

$$x^{(1)} = \frac{([8,10] - [0,1]y^{(0)})}{[7,9]} = \frac{([8,10] - [0,1][0,0])}{[7,9]} = \frac{[8,10]}{[7,9]} = [0.889, 1.429]$$

For $y^{(1)}$:

$$y^{(1)} = \frac{([12,14] - [1,3]x^{(0)})}{[6,8]} = \frac{([12,14] - [1,3][0,0])}{[6,8]} = \frac{[12,14]}{[6,8]} = [1.5, 2.333]$$

2nd iteration: -

$$x^{(2)} = \frac{([8,10] - [0,1]y^{(1)})}{[7,9]} = \frac{([8,10] - [0,1][1.5,2.333])}{[7,9]} = \frac{[5.667,10]}{[7,9]} = [0.630, 1.429]$$

$$y^{(2)} = \frac{([12,14] - [1,3]x^{(1)})}{[6,8]} = \frac{([12,14] - [1,3][0.889,1.429])}{[6,8]} = \frac{[7.713,13.111]}{[6,8]} = [0.964, 2.185]$$

3rd iteration: -

$$x^{(3)} = \frac{([8,10] - [0,1]y^{(2)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.185])}{[7,9]} = \frac{[5.815,10]}{[7,9]} = [0.646, 1.429]$$

$$y^{(3)} = \frac{([12,14] - [1,3]x^{(2)})}{[6,8]} = \frac{([12,14] - [1,3][0.630,1.429])}{[6,8]} = \frac{[7.713,13.37]}{[6,8]} = [0.964, 2.228]$$

4th iteration:

$$x^{(4)} = \frac{([8,10] - [0,1]y^{(3)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.228])}{[7,9]} = \frac{[5.772,10]}{[7,9]} = [0.641, 1.429]$$

$$y^{(4)} = \frac{([12,14] - [1,3]x^{(3)})}{[6,8]} = \frac{([12,14] - [1,3][0.646,1.429])}{[6,8]} = \frac{[7.713,13.354]}{[6,8]} = [0.964, 2.226]$$

5th iteration: -

$$x^{(5)} = \frac{([8,10] - [0,1]y^{(4)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.226])}{[7,9]} = \frac{[5.774,10]}{[7,9]} = [0.642, 1.429]$$

$$y^{(5)} = \frac{([12,14] - [1,3]x^{(4)})}{[6,8]} = \frac{([12,14] - [1,3][0.641,1.429])}{[6,8]} = \frac{[7.713,13.359]}{[6,8]} = [0.964, 2.227]$$

Convergence Conditions: - $|x^{(k+1)} - x^{(k)}| \approx 0$, $|y^{(k+1)} - y^{(k)}| \approx 0$

Since the difference between 4th and 5th iteration is ≈ 0 , the Jacobi method has converged at 5th iteration. Hence the approximate interval solution is $x = [0.642, 1.429]$, $y = [0.964, 2.227]$

2.3. Application Interval Jacobi iteration Method (Three interval linear equations)

In this section, the Interval Jacobi Iteration Method is applied to a 3x3 system of interval linear equations. Compared to the previous example, this problem involves three interval unknowns and demonstrates the applicability of the method to higher-dimensional interval systems.

$$[4, 5]x + [1, 2] y + [1, 1] z = [7, 8] \text{ ——— (1)}$$

$$[1, 2]x + [5, 6] y + [1, 1] z = [8, 9] \text{ ——— (2)}$$

$$[1, 1]x + [1, 1] y + [6, 7] z = [9, 10] \text{ ——— (3)}$$

Rearranging the equation's

$$\text{Eq(1)} \Rightarrow x = \frac{([7,8] - [1,2]y) - [1,1]z}{[4,5]} \quad \text{Eq (2)} \Rightarrow y = \frac{([8,9] - [1,2]x) - [1,1]z}{[5,6]}$$

$$\text{Eq(3)} \Rightarrow z = \frac{([9,10] - [1,1]x) - [1,1]y}{[6,7]}$$

Consider the Initial values: $x^{(0)} = [0, 0]$, $y^{(0)} = [0, 0]$, $z^{(0)} = [0, 0]$

The Iteration formula for Jacobi method is $x^{(k+1)} = \frac{([7,8] - [1,2]y^{(k)}) - [1,1]z^{(k)}}{[4,5]}$

$$y^{(k+1)} = \frac{([8,9] - [1,2]x^{(k)}) - [1,1]z^{(k)}}{[5,6]}, \quad z^{(k+1)} = \frac{([9,10] - [1,1]x^{(k)}) - [1,1]y^{(k)}}{[6,7]}$$

1st Iteration:

$$x^{(1)} = \frac{([7,8] - [1,2]y^{(0)}) - [1,1]z^{(0)}}{[4,5]} = \frac{([7,8] - [1,2][0,0] - [1,1][0,0])}{[4,5]} = \frac{[7,8]}{[4,5]} = [1.4, 2]$$

$$y^{(1)} = \frac{([8,9] - [1,2]x^{(0)}) - [1,1]z^{(0)}}{[5,6]} = \frac{([8,9] - [1,2][0,0] - [1,1][0,0])}{[5,6]} = \frac{[8,9]}{[5,6]} = [1.33, 1.8]$$

$$z^{(1)} = \frac{([9,10] - [1,1]x^{(0)}) - [1,1]y^{(0)}}{[6,7]} = \frac{([9,10] - [1,1][0,0] - [1,1][0,0])}{[6,7]} = \frac{[9,10]}{[6,7]} = [1.28, 1.67]$$

2nd Iteration:

$$x^{(2)} = \frac{([7,8] - [1,2]y^{(1)}) - [1,1]z^{(1)}}{[4,5]} = \frac{([7,8] - [1,2][1.33, 1.8] - [1,1][1.28, 1.67])}{[4,5]} = [0.35, 1.35]$$

$$y^{(2)} = \frac{([8,9] - [1,2]x^{(1)}) - [1,1]z^{(1)}}{[5,6]} = \frac{([8,9] - [1,2][1.4, 2] - [1,1][1.28, 1.67])}{[5,6]} = [0.39, 1.26]$$

$$z^{(2)} = \frac{([9,10] - [1,1]x^{(1)}) - [1,1]y^{(1)}}{[6,7]} = \frac{([9,10] - [1,1][1.4, 2] - [1,1][1.33, 1.8])}{[6,7]} = [0.74, 1.21]$$

3rd Iteration:

$$x^{(3)} = \frac{([7,8] - [1,2]y^{(2)}) - [1,1]z^{(2)}}{[4,5]} = \frac{([7,8] - [1,2][0.39, 1.26] - [1,1][0.74, 1.21])}{[4,5]} = [0.65, 1.72]$$

$$y^{(3)} = \frac{([8,9] - [1,2]x^{(2)}) - [1,1]z^{(2)}}{[5,6]} = \frac{([8,9] - [1,2][0.35, 1.35] - [1,1][0.74, 1.21])}{[5,6]} = [0.68, 1.58]$$

$$z^{(3)} = \frac{([9,10] - [1,1]x^{(2)}) - [1,1]y^{(2)}}{[6,7]} = \frac{([9,10] - [1,1][0.35, 1.35] - [1,1][0.39, 1.26])}{[6,7]} = [0.91, 1.54]$$

4th Iteration:

$$x^{(4)} = \frac{([7,8] - [1,2]y^{(3)}) - [1,1]z^{(3)}}{[4,5]} = \frac{([7,8] - [1,2][0.68, 1.58] - [1,1][0.91, 1.54])}{[4,5]} = [0.46, 1.60]$$

$$y^{(4)} = \frac{([8,9] - [1,2]x^{(3)}) - [1,1]z^{(3)}}{[5,6]} = \frac{([8,9] - [1,2][0.65, 1.72] - [1,1][0.91, 1.54])}{[5,6]} = [0.50, 1.49]$$

$$z^{(4)} = \frac{([9,10] - [1,1]x^{(3)}) - [1,1]y^{(3)}}{[6,7]} = \frac{([9,10] - [1,1][0.65, 1.72] - [1,1][0.68, 1.58])}{[6,7]} = [0.81, 1.45]$$

5th Iteration:

$$x^{(5)} = \frac{([7,8] - [1,2]y^{(4)}) - [1,1]z^{(4)}}{[4,5]} = \frac{([7,8] - [1,2][0.50, 1.49] - [1,1][0.81, 1.45])}{[4,5]} = [0.51, 1.67]$$

$$y^{(5)} = \frac{([8,9] - [1,2]x^{(4)}) - [1,1]z^{(4)}}{[5,6]} = \frac{([8,9] - [1,2][0.46, 1.60] - [1,1][0.81, 1.45])}{[5,6]} = [0.56, 1.55]$$

$$z^{(5)} = \frac{([9,10] - [1,1]x^{(4)}) - [1,1]y^{(4)}}{[6,7]} = \frac{([9,10] - [1,1][0.46, 1.60] - [1,1][0.50, 1.49])}{[6,7]} = [0.84, 1.51]$$

6th Iteration:

$$x^{(6)} = \frac{([7,8] - [1,2]y^{(5)}) - [1,1]z^{(5)}}{[4,5]} = \frac{([7,8] - [1,2][0.56, 1.55] - [1,1][0.84, 1.51])}{[4,5]} = [0.48, 1.65]$$

$$y^{(6)} = \frac{([8,9] - [1,2]x^{(5)}) - [1,1]z^{(5)}}{[5,6]} = \frac{([8,9] - [1,2][0.51, 1.67] - [1,1][0.84, 1.51])}{[5,6]} = [0.53, 1.53]$$

$$z^{(6)} = \frac{([9,10] - [1,1]x^{(5)}) - [1,1]y^{(5)}}{[6,7]} = \frac{([9,10] - [1,1][0.51, 1.67] - [1,1][0.56, 1.55])}{[6,7]} = [0.83, 1.49]$$

7th Iteration:

$$x^{(7)} = \frac{([7,8] - [1,2]y^{(6)}) - [1,1]z^{(6)}}{[4,5]} = \frac{([7,8] - [1,2][0.53, 1.53] - [1,1][0.83, 1.49])}{[4,5]} = [0.49, 1.66]$$

$$y^{(7)} = \frac{([8,9] - [1,2]x^{(6)}) - [1,1]z^{(6)}}{[5,6]} = \frac{([8,9] - [1,2][0.48, 1.65] - [1,1][0.83, 1.49])}{[5,6]} = [0.54, 1.54]$$

$$z^{(7)} = \frac{([9,10] - [1,1]x^{(6)}) - [1,1]y^{(6)}}{[6,7]} = \frac{([9,10] - [1,1][0.48, 1.65] - [1,1][0.53, 1.53])}{[6,7]} \quad z^{(7)} = [0.83, 1.50]$$

$$\text{Convergence Conditions: } - |x^{(k+1)} - x^{(k)}| \approx 0, |y^{(k+1)} - y^{(k)}| \approx 0$$

Since the difference between 6th and 7th iteration is ≈ 0 , the Jacobi method has converged at 7th iteration. Hence the approximate interval solution is: $x = [0.49, 1.66]$, $y = [0.54, 1.54]$, $z = [0.83, 1.50]$

3. INTERVAL GAUSSEIDEL ITERATION METHOD

In numerical analysis, systems of linear equations with uncertain or approximate coefficients frequently arise in engineering, applied sciences, economics, and computational mathematics. Such uncertainties are effectively represented using interval arithmetic, where each coefficient and constant term is expressed as an interval instead of a single real number. The Interval Gauss-Seidel Iteration Method is an extension of the classical Gauss-Seidel method to interval systems of linear equations. This method is used to obtain approximate interval solutions while accounting for uncertainty in the data. Compared with the Interval Jacobi Method, the Gauss-Seidel method generally converges faster because newly computed interval values are immediately used in subsequent calculations within the same iteration.

3.1. Interval Seidel iteration method for system of n- linear equations

Consider the interval system $AX=B$ where A is an interval coefficient matrix, X is the vector of unknown interval variables, and B is the interval constant vector. Each equation is rearranged so that one variable is expressed in terms of the remaining variables. Starting from suitable initial interval approximations, the method computes successive interval solutions iteratively. In the Interval Gauss-Seidel Method, once a new approximation of a variable is obtained, it is immediately substituted into the next equations during the same iteration. This updating strategy improves the convergence rate and reduces the number of iterations required to obtain the desired accuracy. The iterative process is continued until the difference between two consecutive approximations becomes negligibly small. The method is particularly effective for diagonally dominant interval systems, where convergence is generally guaranteed.

Consider the system of equation

$$\begin{aligned} [a_{11}, b_{11}] x_1 + [a_{12}, b_{12}] x_2 + \dots + [a_{1n}, b_{1n}] x_n &= [c_1, d_1] \\ [a_{21}, b_{21}] x_1 + [a_{22}, b_{22}] x_2 + \dots + [a_{2n}, b_{2n}] x_n &= [c_2, d_2] \\ &\vdots \\ [a_{n1}, b_{n1}] x_1 + [a_{n2}, b_{n2}] x_2 + \dots + [a_{nn}, b_{nn}] x_n &= [c_n, d_n] \end{aligned}$$

From above equation of system (1), we have

$$\begin{aligned} x_1 &= \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2 + [a_{13}, b_{13}] x_3 + \dots + [a_{1n}, b_{1n}] x_n] \\ x_2 &= \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1 + [a_{23}, b_{23}] x_3 + \dots + [a_{2n}, b_{2n}] x_n] \\ &\vdots \\ x_n &= \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1 + [a_{n2}, b_{n2}] x_2 \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}] \end{aligned}$$

Let $x_2^{(0)}, x_3^{(0)}, \dots, x_n^{(0)}$ are initial approximations substitute these approximations in the right-hand side of first equation of the above equation

$$x_1^{(1)} = \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2^{(0)} + \dots + [a_{1n}, b_{1n}] x_n^{(0)}]$$

Now substitute the approximations $x_1^{(1)}, x_2^{(0)}, x_3^{(0)}, \dots, x_n^{(0)}$ in the right-hand side of second equation of the above equation

$$x_2^{(1)} = \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1^{(1)} + \dots + [a_{2n}, b_{2n}] x_n^{(0)}]$$

Now substitute the approximations $x_1^{(1)}, x_2^{(1)}, x_3^{(0)}, \dots, x_n^{(0)}$ in the right-hand side of third equation of the above equation. We have

$$x_3^{(1)} = \frac{[c_3, d_3]}{[a_{33}, b_{33}]} - \frac{1}{[a_{33}, b_{33}]} [[a_{31}, b_{31}] x_1^{(1)} + [a_{32}, b_{32}] x_2^{(1)} + \dots + [a_{3n}, b_{3n}] x_n^{(0)}]$$

Continuing this process if we substitute the approximation $x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \dots, x_{n-1}^{(1)}$ in the last equation of system above equation

$$\text{We obtain } x_n^{(1)} = \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1^{(1)} + [a_{n2}, b_{n2}] x_2^{(1)} \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}^{(1)}]$$

Let $x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \dots, x_n^{(k)}$ be the k^{th} approximations of the given system then the $(k+1)^{th}$ approximations are obtained by

$$\begin{aligned} x_1^{(k+1)} &= \frac{[c_1, d_1]}{[a_{11}, b_{11}]} - \frac{1}{[a_{11}, b_{11}]} [[a_{12}, b_{12}] x_2^{(k)} + [a_{13}, b_{13}] x_3^{(k)} + \dots + [a_{1n}, b_{1n}] x_n^{(k)}] \\ x_2^{(k+1)} &= \frac{[c_2, d_2]}{[a_{22}, b_{22}]} - \frac{1}{[a_{22}, b_{22}]} [[a_{21}, b_{21}] x_1^{(k+1)} + [a_{23}, b_{23}] x_3^{(k)} + \dots + [a_{2n}, b_{2n}] x_n^{(k)}] \\ &\vdots \\ x_n^{(k+1)} &= \frac{[c_n, d_n]}{[a_{nn}, b_{nn}]} - \frac{1}{[a_{nn}, b_{nn}]} [[a_{n1}, b_{n1}] x_1^{(k+1)} + [a_{n2}, b_{n2}] x_2^{(k+1)} + \dots + [a_{nn-1}, b_{nn-1}] x_{n-1}^{(k+1)}] \end{aligned}$$

For, $k=0, 1, 2, \dots$ Continue the above process until we get the desired accuracy of the root.

3.2. Application to Interval Gauss Seidel iteration method (Two interval linear equations)

This section demonstrates the application of the Interval Gauss-Seidel Iteration Method to a 2x2 system of interval linear equations.

Consider the system linear equations

$$[7, 9]x + [0, 1] y = [8, 10] \text{ ——— (1)}$$

$$[1, 3]x + [6, 8] y = [12, 14] \text{ ——— (2)}$$

$$[7, 9] > [0, 1], [6, 8] > [1, 3]$$

From equations (1) and (2), write iterative form:

$$\text{Eq(1)} \Rightarrow x = \frac{([8,10] - [0,1]y)}{[7,9]} \quad \text{Eq (2)} \Rightarrow y = \frac{([12,14] - [1,3]x)}{[6,8]}$$

Iteration formula for Gauss-Seidel method:

$$x^{(k+1)} = \frac{([8,10] - [0,1] y^{(k)})}{[7,9]}$$

$$y^{(k+1)} = \frac{([12,14] - [1,3] x^{(k+1)})}{[6,8]}$$

Consider the Initial value is $y^{(0)} = [0,0]$

1st Iteration: -

$$x^{(1)} = \frac{([8,10] - [0,1]y^{(0)})}{[7,9]} = \frac{([8,10] - [0,1][0,0])}{[7,9]} = [0.889, 1.429]$$

$$y^{(1)} = \frac{([12,14] - [1,3]x^{(1)})}{[6,8]} = \frac{([12,14] - [1,3][0.889,1.429])}{[6,8]} = [0.964, 2.185]$$

2nd Iteration: -

$$x^{(2)} = \frac{([8,10] - [0,1]y^{(1)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.185])}{[7,9]} = [0.646, 1.429]$$

$$y^{(2)} = \frac{([12,14] - [1,3]x^{(2)})}{[6,8]} = \frac{([12,14] - [1,3][0.646,1.429])}{[6,8]} = [0.964, 2.226]$$

3rd iteration: -

$$x^{(3)} = \frac{([8,10] - [0,1]y^{(2)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.226])}{[7,9]} = [0.642, 1.429]$$

$$y^{(3)} = \frac{([12,14] - [1,3]x^{(3)})}{[6,8]} = \frac{([12,14] - [1,3][0.642,1.429])}{[6,8]} = [0.964, 2.226]$$

4th iteration: -

$$x^{(4)} = \frac{([8,10] - [0,1]y^{(3)})}{[7,9]} = \frac{([8,10] - [0,1][0.964,2.226])}{[7,9]} = [0.642, 1.429]$$

$$y^{(4)} = \frac{([12,14] - [1,3]x^{(4)})}{[6,8]} = \frac{([12,14] - [1,3][0.642,1.429])}{[6,8]} = [0.964, 2.226]$$

Since the difference between 3rd and 4th iteration is ≈ 0 , The Gauss-Seidel method has converged at 4th iteration. Hence the approximate interval solution is: $x = [0.642, 1.429]$, $y = [0.964, 2.226]$

3.3. Application to Interval GaussSeidel iteration method (Three interval linear equations)

In this section, the Interval Gauss-Seidel Iteration Method is applied to a 3x3 interval system of linear equations. This example illustrates the effectiveness of the method for solving higher-order interval systems involving multiple uncertain variables.

Consider the system linear equations

$$[4, 5]x + [1, 2] y + [1, 1] z = [7, 8]$$

$$[1, 2]x + [5, 6] y + [1, 1] z = [8, 9]$$

$$[1, 1]x + [1, 1] y + [6, 7] z = [9, 10]$$

Rearranging the equation's

$$\text{Eq (1)} \Rightarrow x = \frac{([7,8] - [1,2]y) - [1,1]z}{[4,5]}, \quad \text{Eq (2)} \Rightarrow y = \frac{([8,9] - [1,2]x) - [1,1]z}{[5,6]}$$

$$\text{Eq (3)} \Rightarrow z = \frac{([9,10] - [1,1]x) - [1,1]y}{[6,7]}$$

Consider the Initial values: $y^{(0)} = [0, 0]$, $z^{(0)} = [0, 0]$

Iterative formula for Seidel method:

$$x^{(k+1)} = \frac{([7,8] - [1,2]y^{(k)}) - [1,1]z^{(k)}}{[4,5]}, \quad y^{(k+1)} = \frac{([9,10] - [1,2]x^{(k+1)}) - [1,1]z^{(k)}}{[5,6]}$$

$$z^{(k+1)} = \frac{([9,10] - [1,2]x^{(k+1)}) - [1,1]y^{(k+1)}}{[6,7]}$$

1st Iteration:

$$x^{(1)} = \frac{([7,8] - [1,2]y^{(0)}) - [1,1]z^{(0)}}{[4,5]} = \frac{([7,8] - [1,2][0,0] - [1,1][0,0])}{[4,5]} = [1.4, 2]$$

$$y^{(1)} = \frac{([8,9] - [1,2]x^{(1)}) - [1,1]z^{(0)}}{[5,6]} = \frac{([8,9] - [1,2][1.4,2] - [1,1][0,0])}{[5,6]} = [0.67, 1.52]$$

$$z^{(1)} = \frac{([9,10] - [1,2]x^{(k+1)}) - [1,1]y^{(k+1)}}{[6,7]} = \frac{([9,10] - [1,1][1.4,2] - [1,1][0.67,1.52])}{[6,7]} = [0.789, 1.32]$$

2nd Iteration:

$$x^{(2)} = \frac{([7,8] - [1,2]y^{(1)}) - [1,1]z^{(1)}}{[4,5]} = \frac{([7,8] - [1,2][0.67,1.52] - [1,1][0.789,1.32])}{[4,5]} = [0.53, 1.64]$$

$$y^{(2)} = \frac{([8,9] - [1,2]x^{(2)}) - [1,1]z^{(1)}}{[5,6]} = \frac{([8,9] - [1,2][0.53,1.64] - [1,1][0.789,1.32])}{[5,6]} = [0.57, 1.54]$$

$$z^{(2)} = \frac{([9,10] - [1,2]x^{(2)}) - [1,1]y^{(2)}}{[6,7]} = \frac{([9,10] - [1,1][0.53,1.64] - [1,1][0.57,1.54])}{[6,7]} = [0.83, 1.48]$$

3rd Iteration:

$$x^{(3)} = \frac{([7,8] - [1,2]y^{(2)}) - [1,1]z^{(2)}}{[4,5]} = \frac{([7,8] - [1,2][0.57,1.54] - [1,1][0.83,1.48])}{[4,5]} = [0.49, 1.65]$$

$$y^{(3)} = \frac{([8,9] - [1,2]x^{(3)}) - [1,1]z^{(2)}}{[5,6]} = \frac{([8,9] - [1,2][0.49,1.65] - [1,1][0.83,1.48])}{[5,6]} = [0.54, 1.54]$$

$$z^{(3)} = \frac{([9,10] - [1,2]x^{(3)}) - [1,1]y^{(3)}}{[6,7]} = \frac{([9,10] - [1,1][0.49,1.65] - [1,1][0.54,1.54])}{[6,7]} = [0.83, 1.50]$$

4th Iteration:

$$x^{(4)} = \frac{([7,8] - [1,2]y^{(3)}) - [1,1]z^{(3)}}{[4,5]} = \frac{([7,8] - [1,2][0.54,1.54] - [1,1][0.83,1.50])}{[4,5]} = [0.48, 1.66]$$

$$y^{(4)} = \frac{([8,9] - [1,2]x^{(4)}) - [1,1]z^{(3)}}{[5,6]} = \frac{([8,9] - [1,2][0.48,1.66] - [1,1][0.83,1.50])}{[5,6]} = [0.53, 1.54]$$

$$z^{(4)} = \frac{([9,10] - [1,2]x^{(4)}) - [1,1]y^{(4)}}{[6,7]} = \frac{([9,10] - [1,1][0.48,1.66] - [1,1][0.53,1.54])}{[6,7]} = [0.83, 1.50]$$

Since the difference between 3rd and 4th iteration is ≈ 0 , The Gauss-Seidel method has converged at 4th iteration. Hence the approximate interval solution is: $x = [0.49, 1.66]$, $y = [0.53, 1.54]$, $z = [0.83, 1.50]$. The convergence of iterative methods is analysed using successive approximations. The results demonstrate that interval Gauss-Seidel converges faster than the interval Gauss Jacobi method. Therefore, the Interval Gauss-Seidel Iteration method is an effective, accurate, and computationally efficient technique for solving interval systems of linear equations, particularly for diagonally dominant interval matrices.

5. CONCLUSIONS

In this paper, classical iterative techniques such as the Jacobi and Gauss-Seidel methods were extended using interval arithmetic to solve systems of linear equations with uncertain or approximate data. The study focused on developing Interval Jacobi and Interval Gauss-Seidel iterative methods and analysing their convergence behaviour through numerical examples involving 2×2 and 3×3 interval systems. The use of interval arithmetic provides a reliable framework for handling uncertainties, rounding errors, and computational approximations. By representing coefficients and solutions as intervals, the methods ensure that the exact solution lies within the computed interval bounds. The numerical experiments carried out in this work demonstrate the efficiency and applicability of interval iterative techniques for solving linear interval systems.

The major conclusions obtained from this study are summarized below:

1. The concept of interval arithmetic is highly important in handling uncertainties, rounding errors, and approximate data in numerical computations.
2. Interval arithmetic guarantees that the true solution always lies within the computed interval, making it more reliable than classical numerical methods.
3. The Interval Jacobi and Interval Gauss-Seidel methods successfully extend the classical iterative techniques to interval systems of linear equations.
4. This paper demonstrates that interval iterative methods are highly useful in applications where accuracy, reliability, and error bounds are essential.

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