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Evaluation of Over-speed, Burst Margin and Estimation of Low-cycle Fatigue Life of an Aero Engine Disc

Harinath SP[#1], Sharath Chandra GV [*2], Shreyas PM[#3] , Kumar Gowda [#4]
[#1, 2, 3] *Kshipra simulations Center of excellence and development Banglore-560074, Karnataka, India*
[#4] *Vivekananda Institute of Technology Bangalore - 560074, Karnataka, India*

Abstract: ***Aero engine rotor burst evaluation is one of the most important problems to be taken care off, whenever it comes to designing a turbo machinery disc. The consequences of a fiasco can be intense, since the disc disintegrates into multiple pieces and they are hurled away in all the possible direction at high speeds. Due to high thermo-mechanical loading conditions the disc is subjected to varying degrees of temperature from bore to rim. However, the centrifugal force dominates in the disc which ranges from 85%-90% and the rest can be treated as thermal and gas loads. The challenge lies at designing a disc for off design conditions with their varying loads and duty cycles. In present work evaluation of safety margins and low-cycle fatigue (LCF) estimation of an aero engine disc through classical methods and blending the terminologies with simulation engineering to arrive at a probable interpretation of number of duty cycles is carried out. The methodology compares the fatigue parameters involved in evaluation of disc life. The design tool closely connects the flight certification requirement, namely Flight Readiness Certification, Federal Aviation Administration and European Aviation Safety Agency the regulating agencies for safety in air transportation vehicles. Speed regulations through API and MIL handbook for material specification using finite element analysis approach.***

Keywords: ***Engine rotor burst, Duty cycles of GE CF6-50, over-speed and burst margins, low cycle fatigue, Robinson's and Hallinan's failure criteria's, FRC, Coffin-Manson method.***

NOMENCLATURE

F_c - Centrifugal force in N
M - Mass of the blade in Kg
r_c - Center of Gravity in mm
ω - Angular velocity in rads/s
N - Rotating speed in rpm
P_B - Blade pressure in MPa
N_b - Number of blades = 60
D - Diameter of the disc in mm
t - Thickness of the disc in mm
ϵ_a – Strain amplitude
ϵ_f - Strain at failure
N_f – Number of cycles
c - Ductility factor
CASE 1- Centrifugal loading only
CASE2– Combination of centrifugal force and blade pressure
CASE3– Combination of centrifugal, blade pressure and thermal load

ABBREVIATION

API- American Petroleum Institute
ASTM- American Society for Testing Materials
Mill handbook – Military handbook standards
EASA – European Aviation Safety Agency
FAA – Federal Aviation Administrations
LA – Linear approach
BLA- Bi-linear approach
BIA- Bi-linear isotropic approach
BKA – Bi-linear kinematic approach
OSM – Over-speed margin
BSP- Burst-speed in percentage
CMM- Coffin-Manson Method

I. INTRODUCTION

The two main international authorities who regulate the safety of air transportation, EASA [6] Europe, FAA [6] USA, have set the requirements to be used for civil air transportation engine components. However, military regulations vary from one country to another. Since, they only depend on the internal regulation. The aero engine disc is commonly made of an isotropic material say "INCONEL 718" which is having a particular mechanical property at a specific temperature as per "ASTM" or "MIL" standards. For analysis, duty cycles of a turbofan engine considered is as shown in figure 1[7].The failure criteria's to be employed for designing the disc under the safety regulations and applying classical equations to arrive at a thumb rule are a design challenge. Major disc design criteria are identified with importance on the life limiting design and the application of life estimating techniques.

Design for low-cycle fatigue capability includes consideration of material characteristics, temperature and stress analysis. The evaluation of safety margins and estimation of low-cycle fatigue life in an aero engine disc for design loading conditions (centrifugal force, blade loads and thermal loads) with considering different methods mainly linear and material non-linearity (isotropic and kinematic hardening) is followed. Experimental way of obtaining these safety margins and life of the disc are very expensive and time consuming. Commercial code ANSYS software is employed for the present work. The safety margins and corresponding speeds are obtained by using two different approaches, Robinson's and Hallinan's failure criteria's. The low-cycle fatigue life is estimated for the disc by considering their respective speeds and blending the classical equations using Coffin Manson Method for the present work.

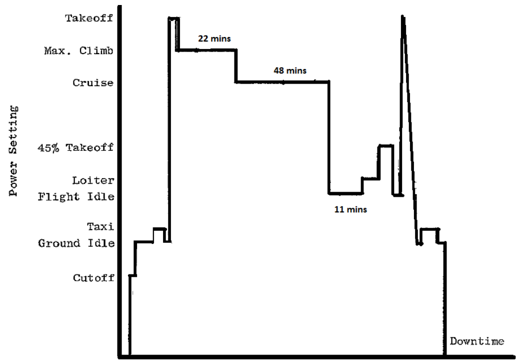

Fig.1 Duty cycles of GE CF6-50 aero engine [7]

The geometry is first optimized using DOE approach while considering topological, design and behaviour constraints. A static analysis is then performed on the obtained geometry considering the various loads acting on it. The mean stress obtained is then used to evaluate the over-speed and burst-margin and the strain values obtained is used for fatigue life estimation. If both the evaluations are within the specified limits, then the design is passed and is safe. If any of the three criteria (Robinson and Hallinan for over-speed and burst margin; and Coffin Manson Method for Fatigue Life Estimation) does not meet the specifications, then the design is again reviewed. The flow diagram for evaluation of over-speed, burst margin and estimation of fatigue life is as shown in figure 2.

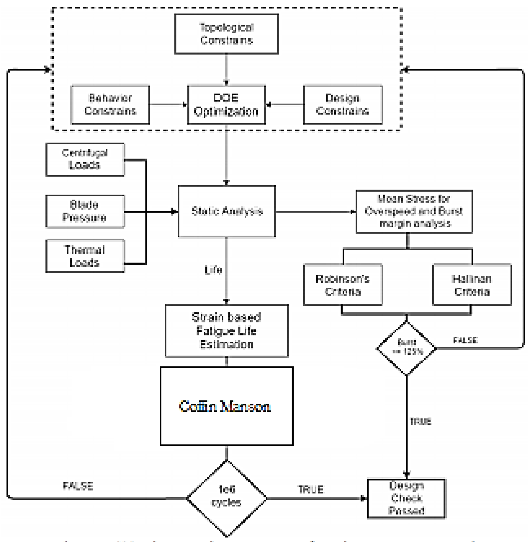

Fig.2 shows the process for the present work

II. SCOPE OF WORK

Gas turbine rotors should pass through design, stress and the quality checks while underdoing certification loads. After the series of checks as per the regulations the major test involves over-speeding margin, burst evaluation, life estimation, fatigue, creep interaction, fracture and failure analysis. However, open literatures reveal individual checks either through mathematical modelling or through experimental work. The modern computers have made it possible to carryout design of experiments employing multi-degrees of freedom analysis carrying multi-design objectives, behaviour constraints to achieve design goal. However, the present work focus is on arriving at optimum disc geometry through DOE considering in-service condition and blends the classical theories with FEA and quickly arrives at commendable methodology which can evolve the design objective in a short duration of time with reliable results and the margin of safety at conservative side.The following are objectives of present work

1. Topological optimization to achieve the design goal.
2. A sensitivity study for material model and its behaviour at in-service condition.
3. 3-D analysis for estimation of over-speed and burst margin evaluation

- *Hallinans approach*
- *Robinsons approach*

4. Estimation of over-speed margin and the burst speed in rotating aero engine disc as per the international authorities for integrity, blending the classical approach with FEA
5. Estimation of low cycle fatigue life
6. Application of 3-D elasto-plastic strain to classical equations to arrive at fatigue life of disc

- *Rain flow counting*
- *Coffin-Manson method*

III. LOAD CONSIDERATIONS

1. ROTATIONAL VELOCITY:
Rotational or angular velocity is defined as the rate of change of angular displacement and is a vector quantity which specifies the rotational speed (angular speed) of an object and the axis about which the object is rotating. Due to this velocity centrifugal force is induced as a body force in the disc.

2. INFLUENCE OF THE BLADES:
They are on the external rim of the disc. The presence of those bodies on the external ring causes an additional traction load in the radial direction, since blades and slots behave like concentrated masses under the effect of the centrifugal field.

Formulas for blade pressure:

$$F_c = m\omega^2 r_c \qquad \text{Eqn.1}$$

$$P_B = \frac{Fc * Nb}{\pi Dt} \qquad \text{Eqn.2}$$

3. THERMAL LOAD:
Loads due to change in temperature are produced by the non-uniform distribution of temperature in the disc under service conditions. The external region of the rotor that is closer to the hot gasses, in fact, is hotter than the internal area. This non-uniform temperature gradient causes a deformation of the material, along with the coefficient of thermal expansion.

LOADING CONDITIONS FOR DISC:

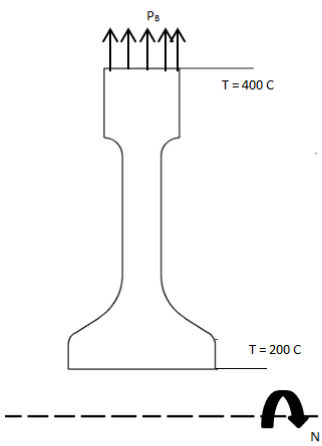

Fig.3 shows the schematic features of the disc

Turbine disc experiences centrifugal loads, thermal loads and blade pressure under operating conditions. The loading conditions of the disc for the present work are classified into three cases: first case is by considering centrifugal force only, second case is combination of centrifugal and blade pressure, third case is by considering all three loads as mentioned above. Figure 3 represents the schematic diagram of disc loading.

IV.MATERIAL

Inconel alloys are corrosion-resistant materials well suited for service in extreme environments subjected to pressure and heat. Since, the aero engine discs are subjected to high centrifugal and thermal loads Inconel is the material preferred for their manufacturing. In the present work, INCONEL 718 is selected which has a gamma double prime strengthened.

TABLE1 SHOWS MATERIAL PROPERTY OF INCONEL718 WHICH IS USED FOR THE PRESENT WORK

Density	8190 kg/m^3
Poison's ratio	0.3
Young's modulus	200 GPa

Tensile strength	725 MPa
Ultimate strength	1035 MPa
Co-efficient of thermal expansion	$13.0*10^{-6}$ K^{-1}

V. DOE (DESIGN OF EXPERIMENTS)

Gas turbine rotors are often subjected to high transient and fluctuating speed loads. Many key parameters from design, material strength, geometry and behaviour play a vital role in reliability and robustness during in-service condition. Sensitivity analysis and DOE become very essential to arrive at an optimum geometry. The design parameters and topological constraints considered for the present work are to arrive at optimum disc geometry through DOE which can be broadly classified in sequence.

1. Design Parameters
2. Behaviour constraints
3. Topological constraints

- DESIGN PARAMETERS
  1. *Allowable hoop and von-Mises stress at the bore is 95% of 0.2% proof stress.*
  2. *Allowable hoop stress at the web is 85% of 0.2% proof stress*
  3. *Allowable radial and von-Mises stress at the web is 80% of 0.2% proof stress.*
  4. *Allowable hoop and von Mises stress at the rim is 67% of 0.2% proof stress*
  5. *Allowable radial growth ≤ specified tolerance of 1 mm*
  6. *Allowable axial growth ≤ specified tolerance of 1 mm.*
- BEHAVIOUR CONSTRAINTS
  1. *The average section stress at cross section of the disc should be within the allowable design limits*
  2. *The average section stress at the cross section should be 10% less than the blade root neck average stress*
  3. *Allowable AWMHS< 72% of 0.2% proof stress at peak temperature in the disc.*
  4. *Allowable AWMHS< 64% of UTS at peak temperature in the disc.*
  5. *Disc burst-speed ≥ 125% of maximum allowable steady state speed for 14000 rpm.*
  6. *Disc over-speed ≥118% of maximum allowable steady state speed for 14000rpm.*

In addition to these standard behaviour constraints on stresses, there are constraints related to the area weighted mean hoop stress (AWMHS) [3] and the area weighted mean radial stress (AWMRS) [3]. These constraints are not readily available in the program like ANSYS and these are the constraints which are used in aero industry. Hence, many non-standard constraints have been incorporated into design rules for the effective evaluation for rotor components.

- TOPOLOGICAL CONSTRAINTS

DOE is performed by utilizing the simple disc 2-D axis symmetry. By executing DOE an optimum design can be obtained with minimum weight and maximum strength. The design parameters which should be obtained by performing DOE is as shown in figure 4.

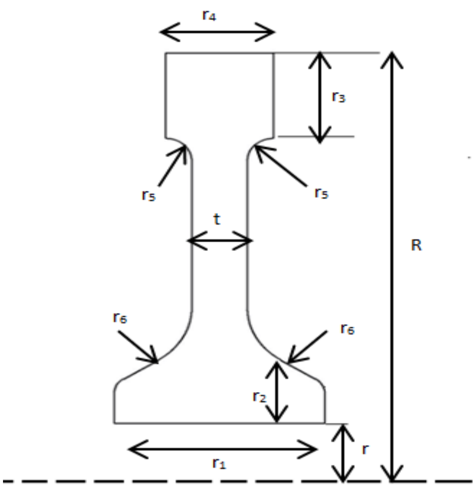

Fig.4 shows the disc parameters

The inner radius 'r' and outer radius 'R' of the disc are constant 50mm and 250mm respectively. From the above obtained parameters range, disc can be designed with an optimum dimension which results in less weight and high strength. By using commercial package ANSYS, a non-linear design of experiments is conducted to arrive at feasible possibility in design space. Based on the constraints the optimum design surface recommended meeting the design goal is considered for further analysis in 3-D. The first cut topology for the given constraints with comparison on the base model is shown in figure 5.

TABLE 2: PARAMETERS RANGE

PARTICULARS	MINIMUM	MAXIMUM
r_1	38	45
r_2	35	43
r_3	25	33
r_4	30	35
r_5	3	5
r_6	4	6
t	12	16

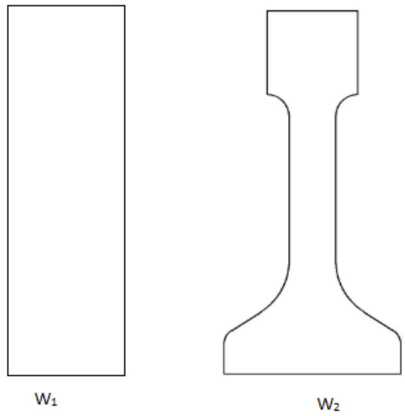

Fig 5 shows the disc section before and after DOE

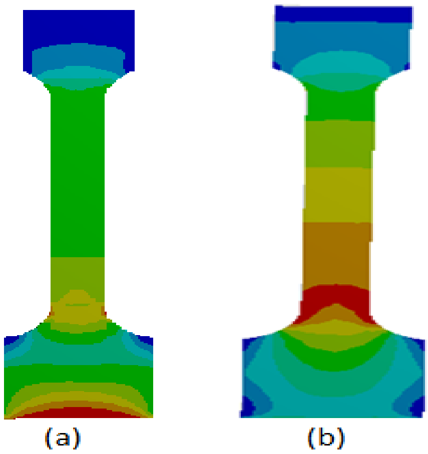

Fig 6 shows the hoop stress (a) and radial stress (b) distrbution in disc sector

The hoop stress (maximum at the disc bore) induced in the disc sector decrease as the length of disc increase from bore to rim. The radial stress (minimum at the bore and maximum in the web) increase in the bore region and decreases in web region as the length increases as shown in figure 6. Since the hoop stress is concentrated at disc bore and plays a major role in disc failure. Hoop stress considered for evaluating the safety margins. The maximum radial growth of the disc obtained from three cases are less than '1 mm' which is below the mentioned constraint.

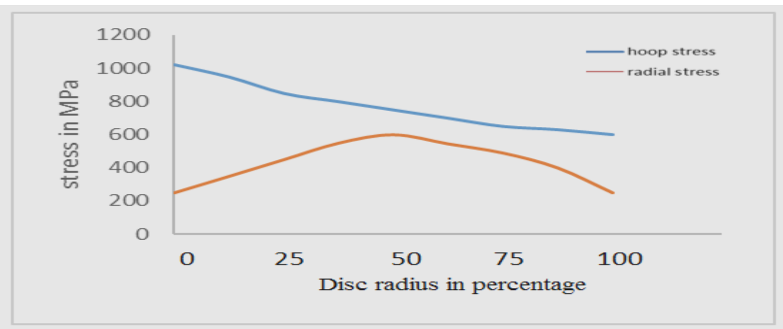

Fig.7 shows variation of stress with respect to the length of the disc sector

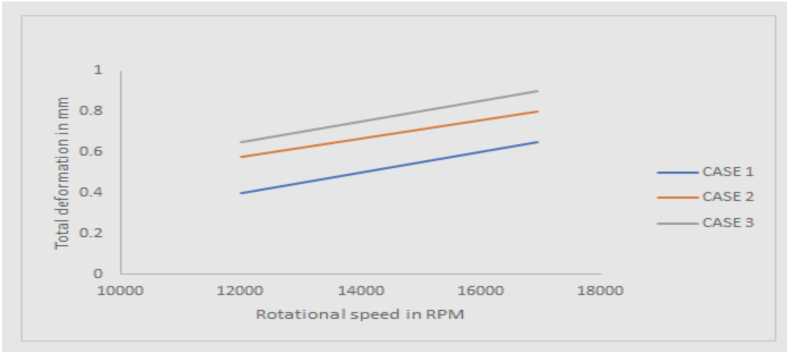

Fig.8 shows the graph plotted for radial growth with change disc speed

VI. EVALUATION OF OVER-SPEED AND BURST MARGINS

Over-speed is a condition where the aero engine disc is allowed or forced to rotate beyond its design limit. The consequences of running the disc too fast vary by engine type, model etc. The duration of over-speed is main factor which speed of disc is dependent upon. In some aero engine discs, even a momentary of over-speed can result in reducing the engine life or even catastrophic failure. The speed at which disc undergoes catastrophic failure is known as burst speed. In the present work, for evaluating the safety margins and burst speed a 3-D model of a disc section is considered as shown in figure 7.

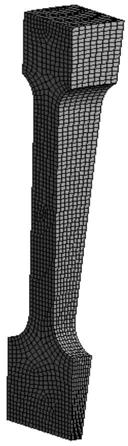

Fig.9 3-D axis- symmetric meshed model using ANSYS

Number of nodes = 37304 ; Number of elements = 7744

The constraints for rotating speed of aero engine turbine disc given by the two international agencies are given below [EASA CS-E 840 AND FAR 33.27] [6]:

a.120% of the maximum permissible rotor speeds associated with any of the ratings of except OEI ratings less than 2 ½ minutes.
b.115% of the maximum permissible rotor speeds associated with any OEI ratings less than 2 ½ minutes.
c. 105% of the highest rotor speed that would result from the failure of the component or system which is the most critical with respect to over-speeding
d. The burst speed is a highest speed that would result from the failure of any component or system in a respective installation of the engine. This failure of a component or system which cannot normally be detected during a routine pre-flight check or during normal flight operation. Gross-yielding approach at critical locations is followed for evaluating the safety margins.

VII. FAILURE CRITERIA for ROTATING DISC

The determination of the burst-speed has brought to the formulation of several theories and criteria that prescribe the procedure to determine the rotational speed that causes failure. Among them, two are used in this work: the Robinson's Criteria also called average hoop stress criterion; and Hallinan's criteria.

ROBINSON'S CRITERIA [7]:

The Robinson criteria is a method which is developed for calculating the burst speed in the hoop mode knowing the ultimate tensile strength, σ_{UTS} and the mean hoop stress, $\sigma_{c\ mean}$. As per this criterion "burst occurs when the mean hoop stress on a disc section becomes equal to the nominal tensile strength of the material, determined from a uniaxial tensile stress".

The mathematical formulation of Robinson criteria,

$$\omega_{burst} = \omega\sqrt{\frac{\sigma_{UTS}}{\sigma_{c,mean}}} \qquad \text{Eqn.3}$$

It is important to know that ultimate strength considered by Robinson Criteria is the engineering stress that differs from true stress. ANSYS applies engineering stress to perform the analysis since; the evolution of safety margins is to be performed by considering true stress. Due to this relation between these stresses is to be performed and utilized for evaluating the safety margins. When passing from engineering stress to true stress, the following relations must be applied:

$$\varepsilon_{true} = \ln(1+\varepsilon_{eng}) \qquad \text{Eqn.4}$$

$$\sigma_{true} = \sigma_{eng}(1+\varepsilon_{eng}) \qquad \text{Eqn.5}$$

From Eqn. 4 and Eqn. 5 a relationship between true stress and engineering stress was obtained. By utilizing these relations, a graph was plotted between stress and strain which is as shown in figure 10.

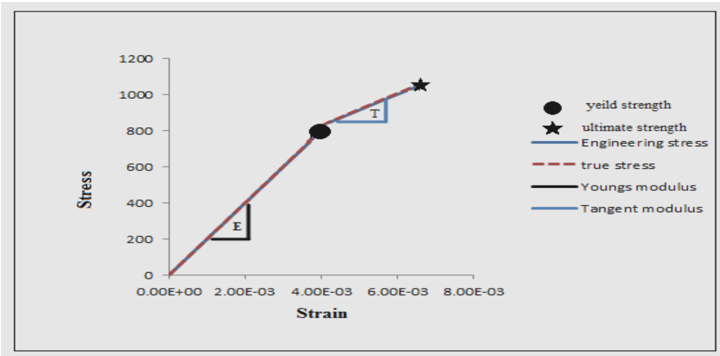

Fig.10 shows the comparison between engginerring stress and true stress which is obtained for this work.

Robinson criteria fail to explain the relation between the true stress and engineering stress. It also fails to explain the effect of maximum hoop stress in the disc failure. Hence this criterion is modified in several ways and one of the forms is Hallinan criteria.

HALLINAN CRITERIA:

This criteria extends the use of the Robinson criteria considering the maximum hoop stress, $\sigma_{c\,max}$ introducing a factor S [7] (it is the ratio of true stress to the engineering stress) to weight the influence of the maximum stress over the mean stress. The ratio S is obtained from the graph figure 10 by comparing true and engineering stress.

The mathematical form of Hallinan criteria

$$\omega_{burst} = 0.95\omega\left[S\left(\sqrt{\frac{\sigma_{UTS}}{\sigma_{c,mean}}} - \sqrt{\frac{\sigma_{UTS}}{\sigma_{c,max}}}\right) + \sqrt{\frac{\sigma_{UTS}}{\sigma_{c,max}}}\right] \qquad \textit{Eqn.5}$$

For this work,

$$S = \frac{\sigma_{TRUE}}{\sigma_{ENG}} - \frac{1038}{1035} \cong 1 \qquad \text{Eqn. 6}$$

Therefore, the equation is reduced to,

$$\omega_{burst} = 0.95\omega\sqrt{\frac{\sigma_{UTS}}{\sigma_{c,mean}}} \qquad \text{Eqn. 7}$$

These two criterions are similar to each other if the value of S is close to 1, such as for discs made of ductile materials. So, in general, the concept of calculating the average stress is associated with the ductility of material and no difference occurs between the two formulations; while if the disc is made of brittle material it is more appropriate to weight the influence of the maximum stress by applying the Hallinan Formula.

1. LA:

CALCULATIONS:

From the behaviour constraints mentioned over-speed $\geq$ 118% of the 100% speed. Hence, for evaluating the safety margins 121% speed (16940 rpm) is considered as per API standards,

$$\text{Over speed Margin} = \sqrt{\frac{\sigma_{yeild}}{\sigma_{c,mean}}} = \sqrt{\frac{725}{577.96}} \qquad \text{Eqn.9}$$

Margin = 1.12

For burst speed,

$$\omega_{burst} = \omega\sqrt{\frac{\sigma_{UTS}}{\sigma_{c,mean}}} = 22400\sqrt{\frac{1035}{1035}} \qquad \text{Eqn.10}$$

$$\omega_{burst} = 22400 \text{ rpm } (160\%)$$

Particulars	Case1	Case2	Case3
OSM	1.12	1.09	1.08
BSP	160%	133%	132%

Similarly, for other cases and methods of analysis similar approach is carried out for calculating the over-speed margin, the burst-speed.

Linearity is just an assumption which simplifies the modelling. Every problem is bi-linear, but solving these models time consumption is more when compare to the linear models. Hence, analysis of linear models is carried out whenever possible. However, the assumptions built into linearity must be considered with every model. It is assumed here that the ultimate stress is within the elastic limit and the there is no yield i.e. the stiffness of the material remains constant in linear analysis which is not real. But, the burst takes place in the disc when the induced stress is equivalent to ultimate stress i.e. the linearity of the material is lost. Therefore, the assumptions made are not applicable for evaluating the burst speed. Hence, the burst speed obtained from this approach is in-valid.

2. BIA:

Particulars	Case1	Case2	Case3
OSM	1.13	1.09	1.08
BSP	156%	128.5%	128%

In isotropic hardening when the material is subjected to loading tension or compression the material deforms linearly till the yield limit. Once the material is crossed the yield limit the material doesn't deforms uniformly. But in this case the deformation of the material takes place by maintaining its center constant. This implies that stress while tension and compression remains same in value but opposite in signs, which is not possible in the real behavior of the material. Hence the obtained burst speeds by this approach are also unacceptable.

3. BKA:

Particulars	Case1	Case2	Case3
OSM	1.12	1.08	1.07
BSP	152%	126%	125.5%

The isotropic model implies that, if the yield strength in tension and compression are initially the same, i.e. the yield surface is symmetric about the stress axes; they remain equal as the yield surface develops with plastic strain. Real metals exhibit some isotropic hardening and some kinematic hardening. In Isotropic hardening the material just hardens until it responds elastically. To fix this, alternative laws i.e. kinematic hardening laws have been introduced. As per these hardening laws, the material softens in compression and thus the yield surface remains the same shape and size but merely translates in stress space, which gives the real behaviour of the material. Hence, this approach gives the real behaviour burst speed obtained are acceptable.

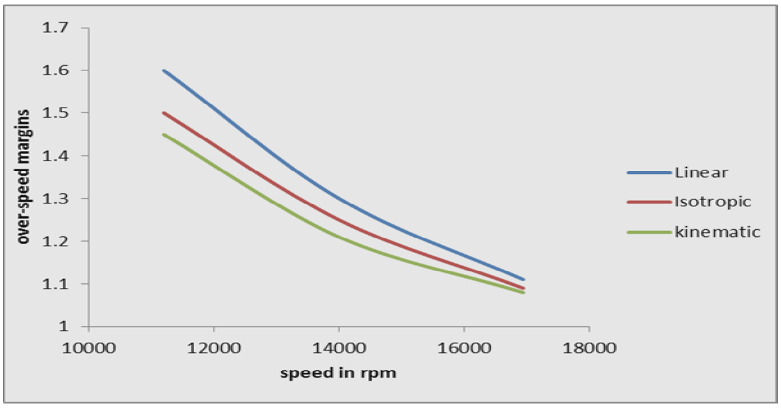

Fig.11 shows OSM for different approachs with respect to change in speed

The evaluated safety margins is plotted in the graph with respect to the change in speed. For over-speed (121% as per API standards) condtion the saftey margins obatined for the disc are in the range of 7-9% which is as shown in figure 11.

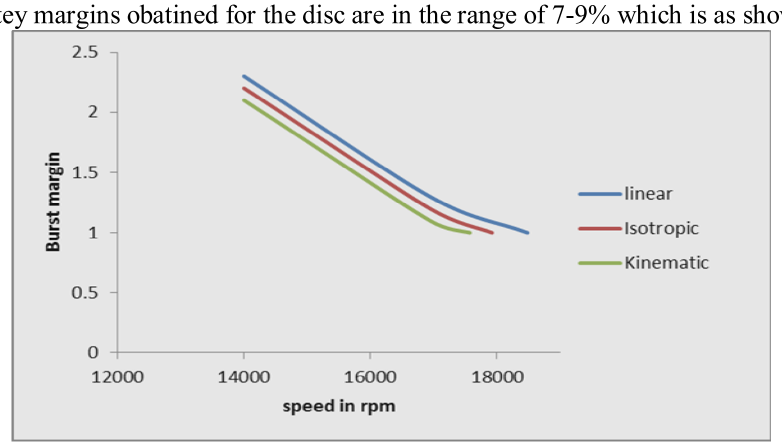

Fig.12 shows burst-margin for different approachs with respect to change in speed

The graph plotted figure 12 is by considering CASE 3 loading condition. The burst-speed obtained for the disc from different approachs is in the range of 125-132% of the operating speed which is as shown in figure 12.

VIII. COMPARISON of BURST SPEED between ROBINSON and HALLINAN CRITERIA

CALCULATION
For this work,
Hallinan criteria = 0.95*Robinson criteria [by using Eqn. (3) & Eqn. (7)]
Therefore,
Burst speed in Hallinan criteria = 0.95* 22400
= 21280 rpm (152%)
Similarly, the above is used to calculate for all other cases.

CASE 1

Particular	Robinson criteria	Hallinan criteria
LA	160%	152%
BIA	156%	148.39%
BKA	152%	144.4%

CASE 2

Particular	Robinson criteria	Hallinan criteria
LA	133%	126.35%
BIA	128.5%	122.07%
BKA	126%	120%

CASE 3

Particular	Robinson criteria	Hallinan criteria
LA	132%	125.4%
BIA	128%	121.6%
BKA	125.5%	119.7%

IX.FATIGUE

ASTM defines fatigue life, N_f, as the number of cycles of specified character that a specimen sustains before failure of a specified nature occurs. In the present work Coffin-Manson method is effectively utilized for estimating the fatigue life of the disc. Rain flow counting is method of isolating small, uninteresting oscillations from the large oscillations, without affecting turning points by the smoothing effect of a filter nor interrupting a large range before it is completed. In fatigue damage calculations, small amplitude ranges can often be neglected as they do not cause the cracks to grow. By following this method, the duty cycles of the aero engine are simplified. By using these simplified duty cycles fatigue life of the disc is estimated.

ESTIMATION OF FATIGUE LIFE:

Estimation of fatigue life cycle is performed by considering the disc at over-speed (121% as per API standards) condition. Coffin-Manson equation [12]: This equation gives a relationship between number of cycles and the total strain (elastic + plastic). In mathematical model the equation can be written as,

$$\epsilon_a = 1.75\,\frac{\sigma_{UTS}}{E}\,N_f^{-.12} + 0.5\,\epsilon_f\,N_f^{-.6} \qquad \text{Eqn 10}$$

It can be reduced to,

$$\frac{\epsilon_a}{2} = \epsilon_f\,2N_f^{c} \qquad \text{Eqn 11}$$

REDUCTION 1:

From the duty cycles of GE- turbo fan shown in figure 1 is reduced to the cycles which shown in figure 13. By employing rain flow counting method the reduced duty cycles are obtained. These duty cycles are obtained by ignoring the cycles of the turbo fan which a have lesser stress amplitudes when compare to other cycles

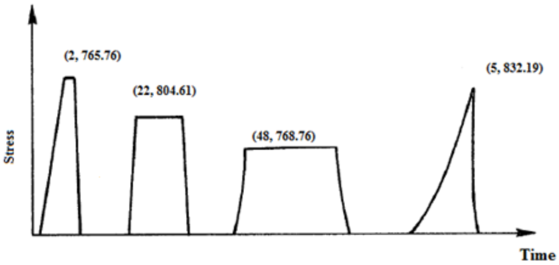

Fig.13 shows the first reduction of duty cycles by applying rain flow counting [7]

Particulars	LA	BLA
Fatigue life cycles	$1.2e^6$ cycles	$1.8e^6$ cycles

REDUCTION 2:

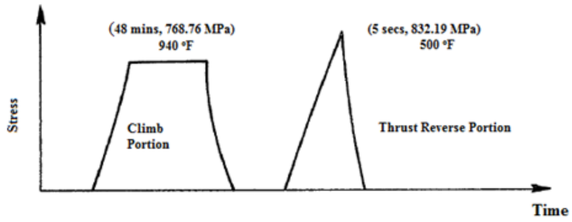

Fig.14 shows the second reduction obtained from rain flow counting of duty cycles [7]

By utilizing rain flow counting method the duty cycles can further be reduced. The duty cycles shown in figure 13 are further reduced to duty cycles as shown is figure 14. This reduced duty cycles is obtained by considering the peak stress induced and the stress with maximum duration.

Particulars	LA	BLA
Fatigue life cycles	$1.2e^6$ cycles	$1.8e^6$ cycles

X.CONCLUSION

The sensitivity analysis and design checks conducted through design of experiments by blending the classical equations and methodologies has thrown light on various aspects including the safety margins evaluation and keeping design parameters which is responsible for mechanical integrity is achieved from present study. Considering the uncertainty at design stage and the off-design condition of safety margins at operating and off design condition is successfully carried out. Classical theories on fatigue are discussed in length to estimate the fatigue life of the component under critical loading conditions to achieve appreciable results. Successfully the safety and integrity as per regulatory bodies controlling the safety of engine components were achieved.

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