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# DESIGN of EXPERIMENTS THROUGH NUMERICAL APPROACH to SOLVE MULTISTAGE ROTOR FAILURE

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**Abstract** - With the demand for the thrust-to-weight ratio of current generation aero-engines continuously increasing the need for pushing advanced engine-materials to operate even in the non-linear regime satisfactorily, becomes mandatory. Critical engine-components need to meet the conflicting requirements of stiffness, strength, fatigue, creep and impact and operate in high speed, pressure and temperature environments, with minimum component weight and reliability associated with aero-engines. Scientific approach such as Design of experiments is now a commonly used tool across industries to optimize component design. Advance Design optimization tool includes design objectives, variables, behavioural constraints for parametric optimization of design. Design of experiments pay way for flexibility in machine design adopting computational techniques in recent years. Sensitivity analysis, robustness through parametric evaluation aiming at design objectives, variables, behavioural constraints for parametric optimization of design. Optimum design surface method has made possible with available solver to arrive at best possible probable optimized design. The main aim of this paper is to model for a multistage drum type rotor coupling of a gas turbine engine typical two stage of a LP turbine. Linear design-optimization tool is used successfully to reduce the weight of a turbine rotor disc, without violating the geometrical constraints imposed and meeting all the design goals. Cases where 10 per cent weight reductions have been achieved in discs, through axi-symmetric analysis, with considerable saving in analysis time, are presented in this paper. Multi-stage turbine disc and coupling arm optimization cases are presented and discussed in full length.
**Keywords:** Campbell Diagram, Design of Experiments, FEA, Optimization, Rotor Coupling, Structural Integrity, Vibration

## I. INTRODUCTION

The functional necessities and features of flexing coupling is to transmit rated torque without undergoing buckling, permanent deformation i.e. to possess with high torsional rigidity. However, under conditions of misalignment, flexing coupling element must have necessary flexibility to accommodate these situations without inducing excessive force and moment on shoulder shaft, bearing and bolts. Misalignment is compensated using laminated disc sets. Both of the requirements should be achieved by maintaining stress levels which are safely in the range of fatigue limit of flexing material. Metal-flexing coupling is identified to show irregular large-amplitude vibrations in axial direction when excited at natural frequency of coupling.

Considering various loads acting upon the disc and variation of the loads with respect to time are the factors that add to the complexity of turbine disc design. Weight of disc plays a vital role in improving efficiency of the gas turbine [1]. Hence, allows component to operate under plastic zone assuring the safety of the component with design limits. In present work an attempt is made to understand design criteria's used for the design of gas turbine disc running at speed of 12000 RPM [2] and operating at a temperature of 500°C. The finite element analyses were carried out to check the mechanical and structural integrity of the disc in a systematic order using the commercially validated FE package software.

## II.ANALYSIS OBJECTIVES

Rotating engine components require high stiffness and low weight. The present requirement at which the designers are focused is at arriving high weight to thrust ratio. Often these rotors attain over-speed and vary with thermal gradients leading to peak strain accumulation at critical location flange bolts are subjected to cyclic loading along with pre-tensions [3]. The possibility of losing the pre-tension in the blots due to excessive plastic strain accumulation during over-speeding reduces the fatigue life loading leading to fatigue creep interaction.
The objective of present work is to look into the design considerations to be accounted for during the design of a gas turbine rotor. Design of gas turbine rotor with defined geometrical constraints,

1. Topological optimization to achieve the design goal.
2. A sensitivity study for material model and its behaviour at in-service condition [4]
3. Static and Bi-linear kinematic analysis
4. Flange and bolt contact analysis
5. Validate numerical and simulation results of stress [5]
6. Run out analysis of bolts.

## III. OPTIMIZATION METHODOLOGY with LOADING CONDITIONS

This section presents the strategy adopted to achieve the optimization of turbine rotor disc. Schematic of the proposed strategy is shown in Figure 1. Proposed strategy includes two distinct methodologies named as Simulator and Optimizer integrated to achieve optimum position of heat loop. The Simulator comprises of FEA model and Optimizer comprises of Design of Experiments (DOE) based optimization technique

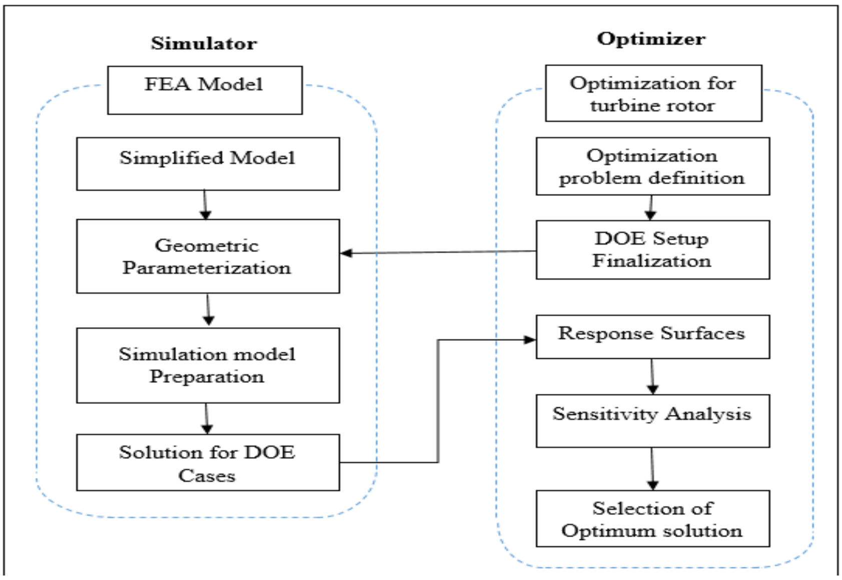



Figure 1: Schematic for proposed strategy

A rotating hot section component in a turbine engine generally subjected to surface loads, centrifugal loads and thermal loads. In the present work surface loads (aerodynamic) loads are neglected instead the blade load is considered. The centrifugal loads arising from the mass of the rotor disc is usually most crucial acting on the turbine disc. This load is determined through the FE calculation after defining the axis of symmetry, the rotational speed and the disc material properties. In the present work, for analysis the operational turbine speed is applied in stepped loading as shown in figure 2, where it also shows variation of stress in disc with respect loading cycles.The main methodology adopted to study structural integrity of rotor coupling bolting system is when respective bolts failed i.e., For Run out analysis, when 3,6,9,12 clock position in rotor coupling bolts failed. [6]

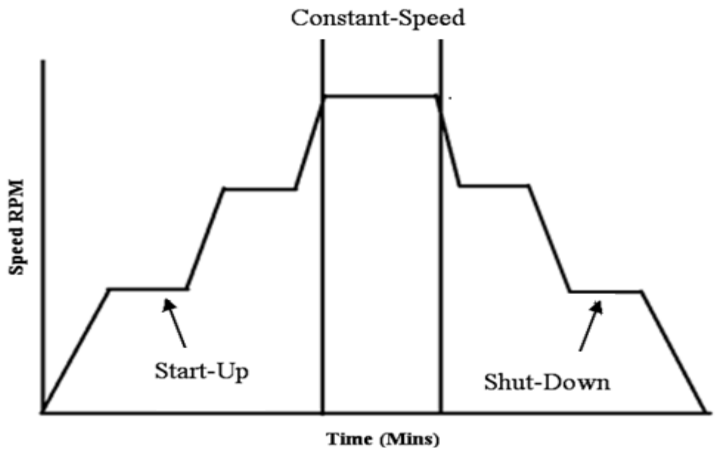



Figure 2: Flanged coupled rotor assembly loading Pattern

## IV.PARAMETRIC BASE LINE MODEL: DESCRIPTION of MODEL

A cyclic symmetry sector of an integrated bladed disk assembly is considered for present analysis. The model of three blades with disk sector of $18^0$ is modelled using the commercial software. The FE model of bladed disk sector is as shown in Figure 3. For the present analysis, higher order elements are considered to generate the finite element model. A matching node pattern is maintained at the blade and disk faces, where the load transfers between blade and disk takes place. The blade and disk are considered to be made of similar material.

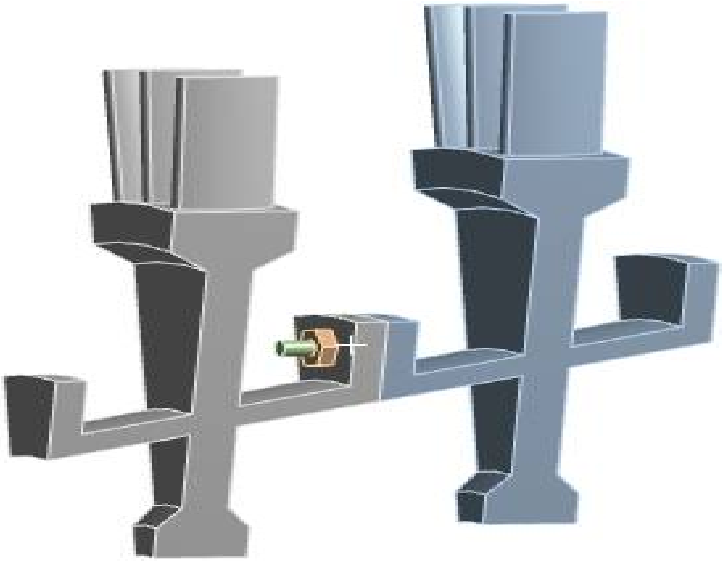


Figure 3: 3-D axis symmetric base model of rotor coupling

## V.MATERIAL MODEL

For present work material of disc is considered as INCONEL 718 [10]. This alloy is a hardened nickel-base alloy with good strength, ductility and fracture toughness over a range of -50 to 600 $^oC$ [2]. These properties along with good weldability and formability account for wide use in aerospace applications. The mechanical properties are tabulated below.

Table (1) shows material property of INCONEL718 [7]

| | |
|---|---|
| Density | 8190 $kg/m^3$ |
| Poisson's ratio | 0.3 |
| Young's modulus | 209 GPa |
| Tensile strength | 1035 MPa |
| Co-efficient of thermal expansion | $13.0*10^{-6}$ $K^{-1}$ |



## VI. NUMERICAL VERIFICATION for ROTOR STRESS

The present methodology assumes that total mechanical strain induced is the sum of elastic and plastic strain, [8]

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p}$$

From Hooke's law,

$$\varepsilon_{ij}^{e} = \frac{1+v}{E}\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij}$$

The plastic strain is assumed to be given by Hencky's total deformation theory

$$\varepsilon_{ij}^{p} = \left(\sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij}\right)$$

From the above two equations total strain is given by,

$$\varepsilon_{ij} = \left(\frac{1+v_{eff}}{E_{eff}}\right)\sigma_{ij} - \frac{v_{eff}}{E_{eff}}\sigma_{kk}\delta_{ij}$$

Where the effective strain values,

$$E_{eff} = \frac{3E}{3+2E}, \quad v_{eff} = \frac{3v+E}{3+2E}$$

For further process disc is assumed to consist of a number of uniform strips as shown in figure 4, each having boundary conditions of rotating disc with internal and external pressure.

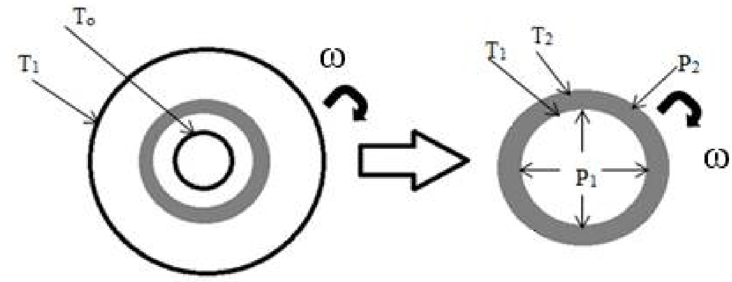



Figure 4: Discretization of rotor disc

Figure 4 represents a typical discretized annular strip of a disc. The solution in displacements form as a function of the radius 'r', for each strip is well-known Lames solution given below, [8]

$$u(r) = \left[m_1 r + \frac{m_2}{r}\right] + \left[n_1 r + \frac{n_2}{r} - \frac{1-v^2}{8E}\rho\, r^3\omega^2\right] + \left[\alpha(1+v)r\, j(r) + \alpha(1-v)Ir + \alpha r_1^2(1+v)\frac{I}{r}\right]$$

where,

$$m_1 = \frac{1-v}{(r_1^2 - r_2^2)}(p_2 r_1^2 - p_1 r_1^2)$$

$$m_2 = \frac{r_1^2 r_2^2(1+v)}{E(r_1^2 - r_2^2)}(P_2 - P_1)$$

$$n_1 = \frac{(1-v)(3+v)}{8E}\rho\omega^2(r_1^2 + r_2^2)$$

$$n_2 = \frac{(1+v)(3+v)}{8E}\rho\omega^2(r_1^2 + r_2^2)$$

$$I = \frac{1}{r_1^2 - r_2^2}\left[\frac{r^2}{2}\left(A\left\{ln\left(\frac{r}{r_2}\right) - \frac{1}{2}\right\}\right) + T_2\right]_{r_1}^{r_2}$$

$$J(r) = \frac{1}{r^2}\left[\frac{r^2}{2}\left(A\left\{ln\left(\frac{r}{r_2}\right) - \frac{1}{2}\right\}\right) + T_2\right]_{r_1}^{r}$$

$$A = \frac{T_1 - T_2}{ln\left(\frac{r_1}{r_2}\right)}$$

The inner and outer displacements of each strip is related to its angular velocity, inner, outer parameters and temperatures in the following form

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{-1}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{-1}\begin{Bmatrix} \Omega_1 \\ \Omega_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{-1}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$$



where,

$$C_{11} = \frac{r_1}{E(r_1^2 - r_2^2)}[(1+v)r_2^2 + r_1^2(1-v)]$$

$$C_{22} = \frac{r_2}{E(r_1^2 - r_2^2)}[(1-v)r_2^2 + r_1^2(1+v)]$$

$$C_{12} = \frac{2r_1 r_2^2}{E(r_1^2 - r_2^2)}, \qquad C_{21} = \frac{2r_2 r_1^2}{E(r_2^2 - r_1^2)}$$

$$\Omega_1 = \frac{\rho\omega^2 r_1}{8E}[6r_2^2 + 2r_1^2 + 2vr_2^2 - 2vr_1^2]$$

$$\Omega_2 = \frac{\rho\omega^2 r_2}{8E}[2r_2^2 + 6r_1^2 + 2vr_1^2 - 2vr_2^2]$$

$$\theta_1 = 2\alpha I r_1$$

$$\theta_2 = \alpha(1+v)r_2 J(r_2) + \alpha I r_2\left[(1-v) + (1+v)\frac{r_1^2}{r_2^2}\right]$$

To obtain behavior of disc, the isolated strips are assembled to form a system to linear equations,

$$[C]^{-1}\{U\} = \{P\} + [c]^{-1}[\{\Omega\} + \{\theta\}]$$

The hoop and radial stress are derived form,

$$\sigma_{rr} = \left[A_1 - \frac{A_2}{r^2}\right] + \frac{3+v}{8}\rho\omega^2\left[r_2^2 + r_1^2 - \frac{r_1^2 r_2^2}{r^2} - r^2\right] + \alpha E\left[-J(r) + I\left(1 - \frac{r_1^2}{r_2^2}\right)\right] \text{ [5]}$$

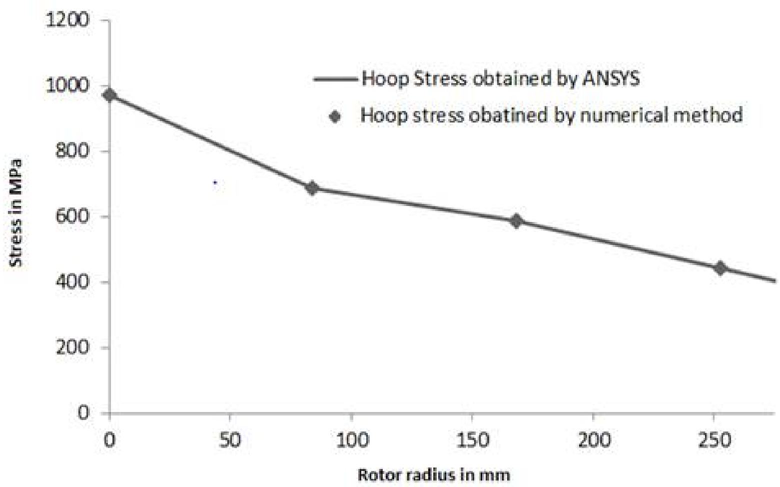
$$\sigma_{\theta\theta} = \left[A_1 + \frac{A_2}{r^2}\right] + \frac{3+v}{8}\rho\omega^2\left[r_2^2 + r_1^2 + \frac{r_1^2 r_2^2}{r^2} - \frac{1+3v}{3+v}r^2\right] + \alpha E\left[-J(r) + I\left(1 - \frac{r_1^2}{r_2^2}\right)\right] \text{ [5]}$$

Where,

$$T(r) = A\ln\left(\frac{r}{r_2}\right) + T_2,$$

$$A_1 = \frac{r_2^2 P_2 - r_1^2 P_1}{r_1^2 - r_2^2}$$

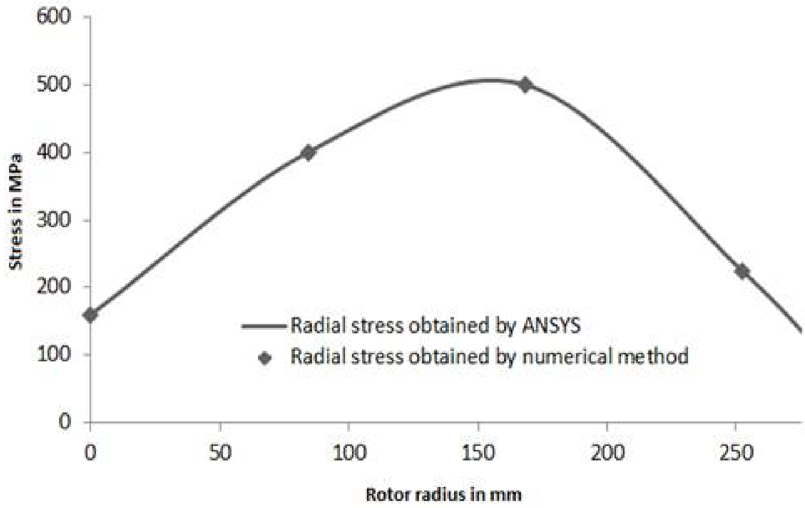
$$A_2 = \frac{(P_2 - P_1) r_2^2 r_1^2}{r_1^2 - r_2^2}$$




(a)






(b)

Figure 5: Comparison of (a) hoop stress and (b) radial stress obtained from numerical and ANSYS

From the above figure 5 (a) and (b) it is observed the results obtained from numerical and simulations method are matching with an error of less than 8%. The hoop stress is more dominating than radial stress.

## VII. SIMULATIONS RESULTS for BASELINE MODEL

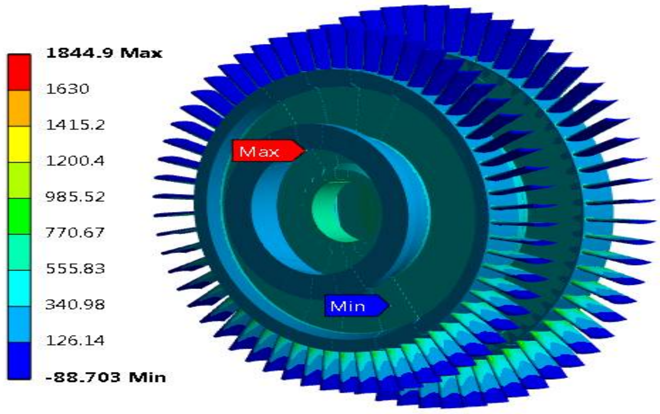



Figure 6: Maximum principle stress in flange coupled rotor

The modes of failure in a rotating disc is mainly due to tensile loading resulting with centrifugal load along with transient temperatures varying from bore to rim. For the given material only tensile loading allowable design stress of 1035 MPa is considered for the maximum temperature of 600$^0$C as a conservative approach in the present work. The tensile stress developed in the disc, rotor arms, bolt holes decide the fatigue life of the component. As a result, figure 6 represents maximum principle stress in the rotor along with the bolt assembly is 1844.9 MPa which exceeds the design limit of rotor.

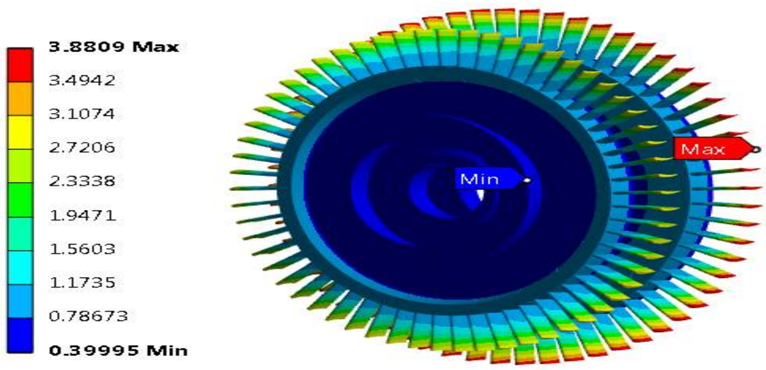



Figure 7: Radial deformation in flange coupled rotor



The radial growth at the flange region is 3.88 mm as shown in figure 7 which is higher than allowable growth of 1.14mm. For a given poisson's ratio of 0.3 for metallic structures the normal thumb rule is radial deformations to be limited to 1.14mm [9]. Hence, the deformation is well within the limit of operating conditions. Hence, topological optimization is followed to arrive at an optimum flange coupled rotor which is explained in succeeding sections.

## VIII.DOE (DESIGN of EXPERIMENTS)

Gas turbine rotors are often subjected to high transient and fluctuating speed loads. Many key parameters from design, material strength, geometry and behavior play a vital role in reliability and robustness during in-service condition. Sensitivity analysis and DOE become very essential to arrive at an optimum geometry. The design parameters and topological constraints considered for the present work are to arrive at optimum disc geometry through DOE which can be broadly classified in sequence.

1. Design Parameters
2. Behavior constraints
3. Topological constraints

### A. Design Parameters

1. At the Bore, permissible hoop and equivalent stress is 95% of 0.2% proof stress.
2. At the web, permissible hoop stress is 85% of 0.2% proof stress
3. At the web, permissible radial and equivalent stress is 80% of 0.2% proof stress.
4. At the rim, permissible hoop and equivalent stress is 67% of 0.2% proof stress
5. Axial and radial growth ≤ specified tolerance i.e., 1 mm

### B. Behaviour Constraints

1. The average section stress at cross section area of disk must not exceed allowable design limits
2. Average cross sectional stress should not cross the limit of 10% of blade root neck average stress
3. Permissible AWMHS< 72% of 0.2% proof stress at peak temperature in the disk. [9]
4. Permissible AWMHS< 64% of UTS at peak temperature in the disc. [9]
5. Disc burst-speed ≥ 125% of maximum permissible steady state speed for 12000 rpm.
6. Disc over-speed ≥118% of maximum permissible steady state speed for 12000rpm.

These constraints are not readily available in the program like ANSYS and these are the constraints which are used in aero industry.

### C. Topological Constraints

DOE is performed by utilizing the simple disc 3D axis symmetry [10]. By executing DOE, an optimum design is obtained with minimum weight and maximum strength. The design parameter obtained for performing DOE are as shown in figure 8.

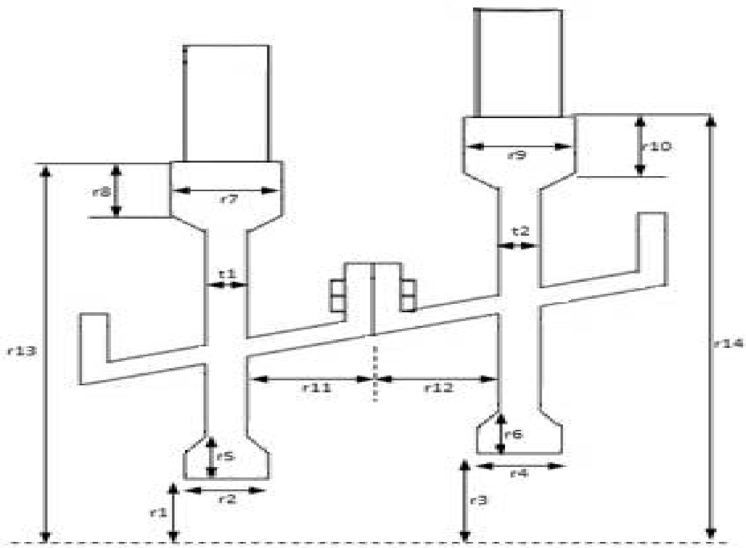



Figure 8: Disc parameters

The radius 'r13' and radius 'r14' of the disc are constant 300 mm 325 mm and inner radius respectively. From the above obtained parameters range, disc can be designed with an optimum dimension which results in less weight and high strength. By using commercial FE package, a non-linear design of experiments is conducted to arrive at feasible possibility in design space.



Based on the constraints the optimum design surface recommended meeting the design goal is considered for further analysis in 3-D. The first cut topology for the given constraints with comparison on the base model is shown in figure 8.

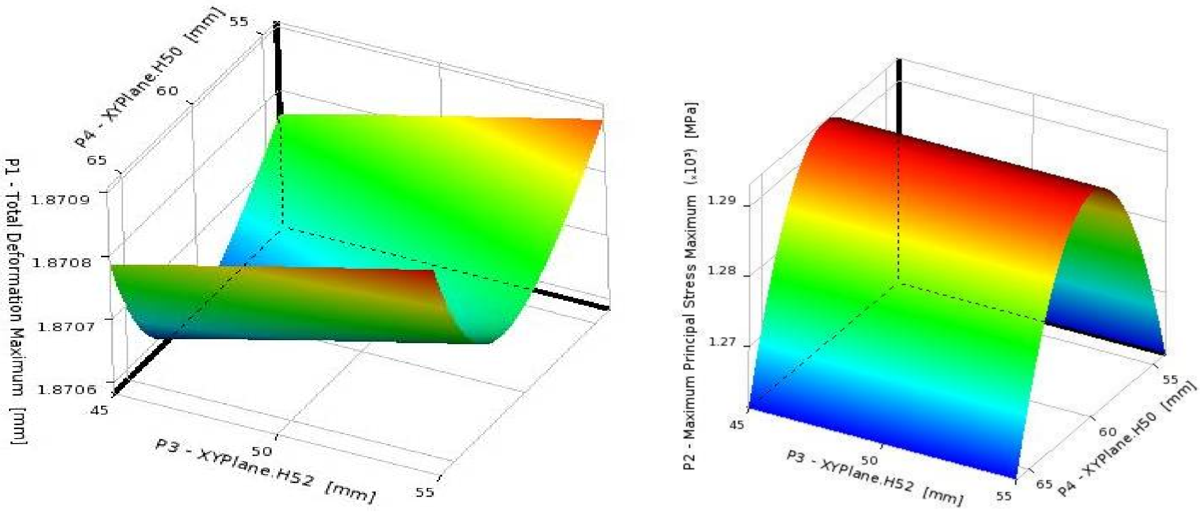



Figure 9: Variation of deformation and maximum principle stress with respect to design points

A numerical optimize is carried out for the base-line model by utilizing axi-symmetry method. In figure 9 it is observed the variation of radial deformation and maximum principle stress with respect to the design parameters. The 3-D graph depicts the best possible combination of design points for an optimum flanged disc assembly. Figure 10 shows the best possible outcome by following numerical design optimization method and further two configuration i.e, base-line model (configuration 1) and optimized model (configuration 2) is considered for comparative study.

## 1XFINITE ELEMENT MODEL of OPTIMIZED FLANGED COUPLED ROTOR

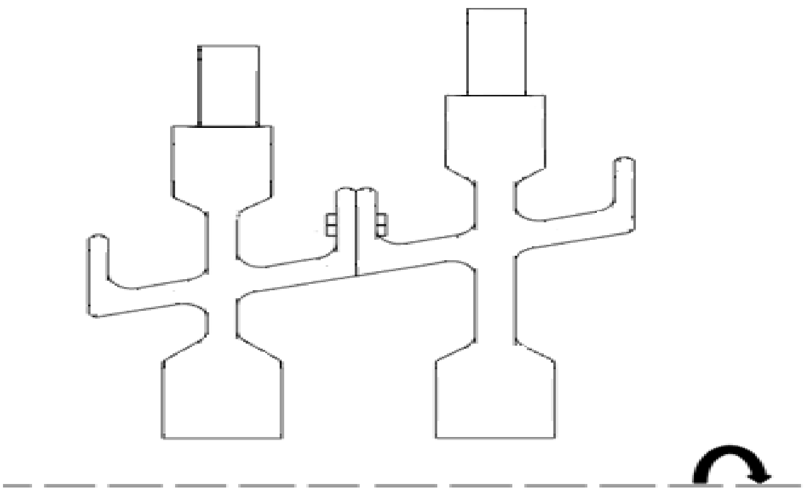


Figure 10: 3-D axis symmetry and mathematical model of optimized rotor coupling

## X.COMPARISON of RESULTS BETWEEN BASE and OPTIMIZED MODEL

In this section for the given loading condition the stresses resulting from the physical model employing different methodologies to simulate the vilest loads coming on the component is discussed in full length the preliminary design consideration as employed as stress checks to achieve the mechanical integrity in rotor coupling. Normally the methods adopted in the structural evaluation in aero engine rotors. Nonlinear /bilinear kinematic is employed when high thermal gradient and geometrical non- linearity's, material non-linearity's are directly considered in couplings due to eccentric bolt holes and possibility of plastic stains.



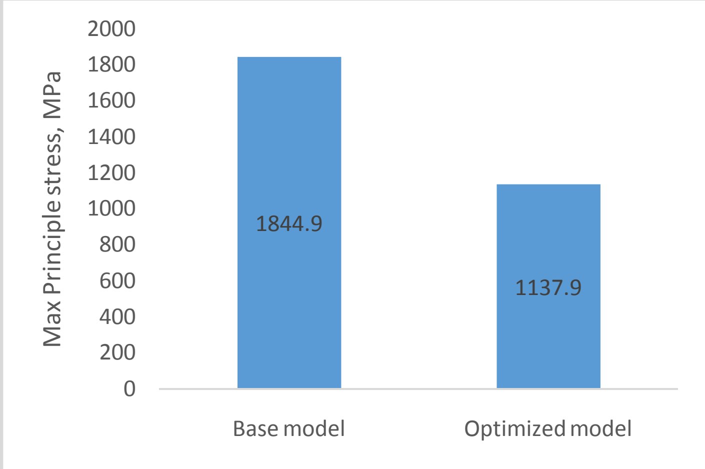



Figure 11 Maximum Principle Stress for optimized rotor coupling

Since in the modes of failure in a rotating disc is mainly due to tensile loading resulting with centrifugal load along with transient temperatures varying from bore to rim. For the given material only tensile loading allowable design stress of 1035 MPa is considered for the maximum temperature of 600$^0$C as a conservative approach in the present work. The tensile stress developed in the disc, rotor arms, bolt holes decide the fatigue life of the component. The peak stress resulted from the loading condition will decide the number of start-up and shut down cycles required for crack initiation. Zero max zero loading condition is supposed to be considered for fatigue loads coming on a rotor, bolts, bolt holes and rotor assembly. It is also anticipated due to the tensile nature the maximum principle plane will contain the highest tensile stress. As a result, figure 11 shows the maximum principle stress in the rotor along with the bolt assembly is 1137.9 MPa however these stressis localized stress limited in the bolt zone and at the sharp edges which are specious in nature concentrated due to bolt hole. However, the average stress at any section of the disc from bore to rim is ranging between 450 to 580 MPa against the design stress of 1035 MPa.

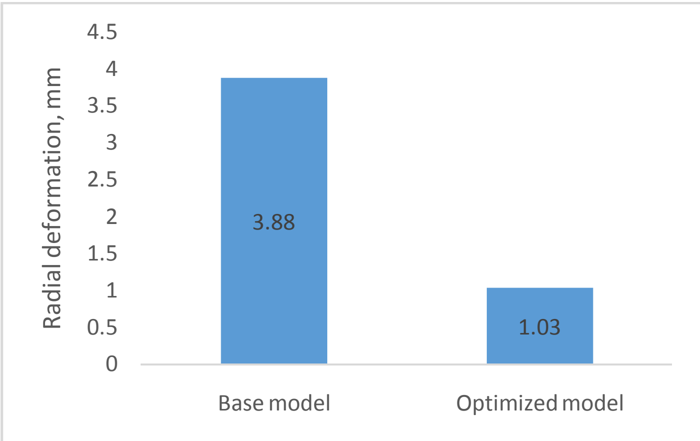



Figure 12: Radial deformation for Base model and optimized rotor coupling

The radial deformation at the flange region is 0.97 mm which is less than the allowable growth of 1.14mm. For a given poisson's ratio of 0.3 for metallic structures the normal thumb rule is radial deformations to be limited to 1.14mm. Hence, the deformation is well within the limit at operating conditions.

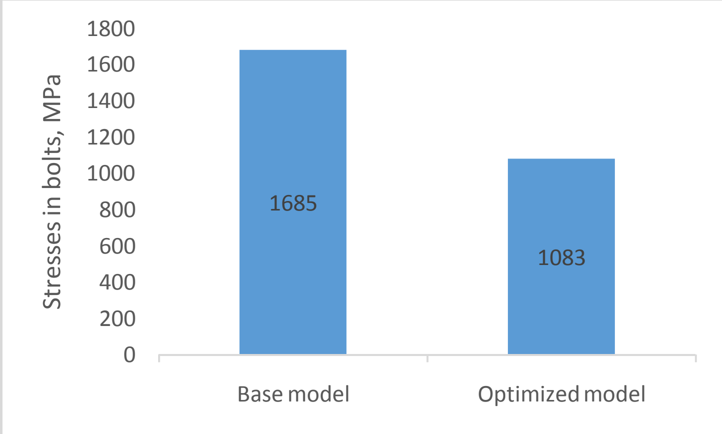



Figure 13: Maximum Principle Stress for Base model and optimized bolts

The bolt pretention is provided in the bolts to increase the contact stress in the flange retaining the tensile load parallel to the axis of the rotor. Due to centrifugal pull, each bolt is subjected to shear loading since each stage of the rotor tries to radially grow outward resulting in combination of bolt bending and shear. The first principle stress in bolt indicates the shear loading dominance in the bolt and the equivalent stress indicates the stress intensity due to bolt bending. The bolt crushing is indicated by studying the stress intensity across the bolt region through equivalent von misses stress. Figure 13 indicates the maximum equivalent stress developed in the bolt in the operating range is 1083 MPa. However, the stress ranges between 850 MPa to 925 MPa which is well within the design limits and the design is safe.

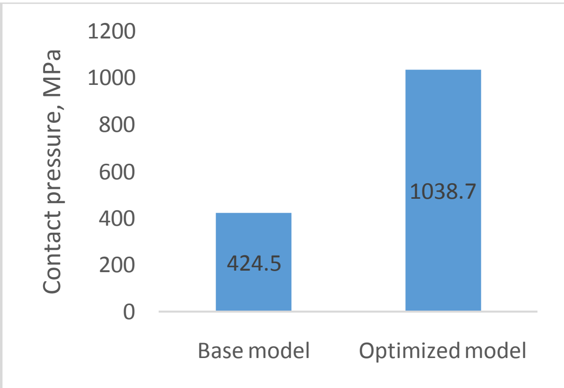



Figure 14: Comparison of contact pressure for two configuration

In the above figure 14 it is observed that contact pressure in optimized model where maximum magnitude is 1038.7 MPa where as in the base model maximum contact pressure is 424.5 MPa.

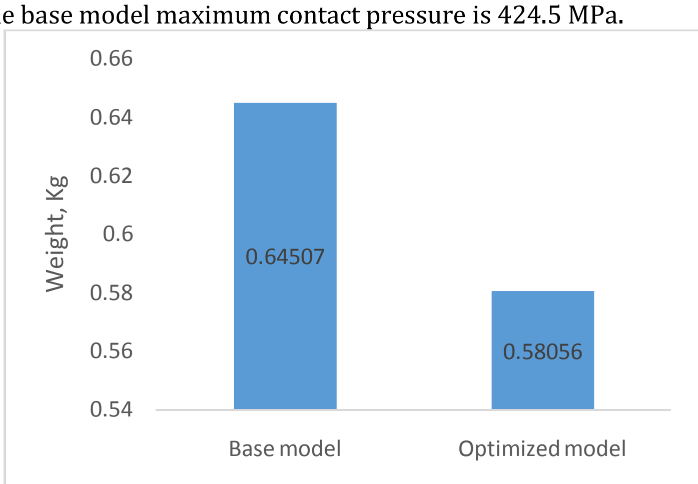
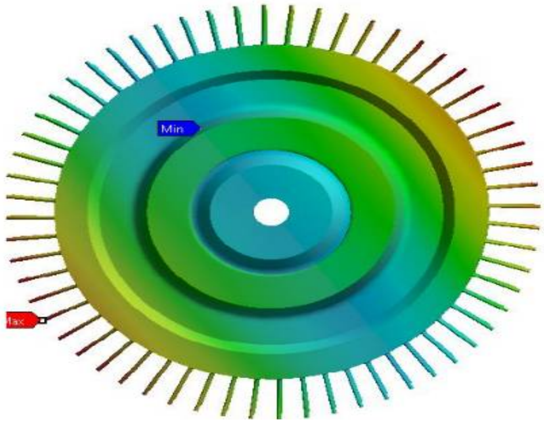



Figure 15: Comparison of weight reduction for two configurations

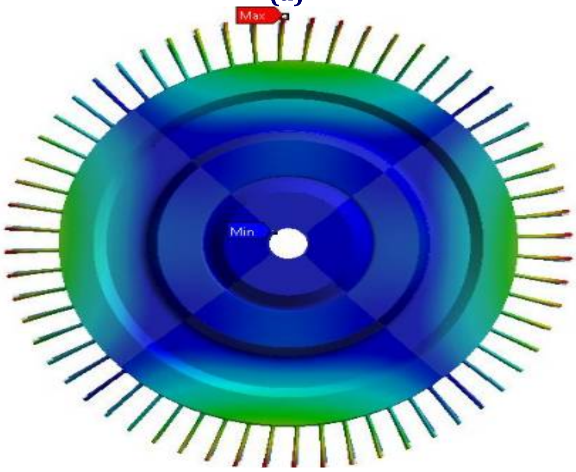
In the above figure 15 it is observed that final weight of optimized model is reduced by 10% of initial base-line model whereas initial weight of baseline model is 0.64507 Kg and weight of optimized model 0.58056 Kg.






**(a)**




**(b)**

Figure 16(a): 1st Mode
Figure 16(b): 2nd Mode

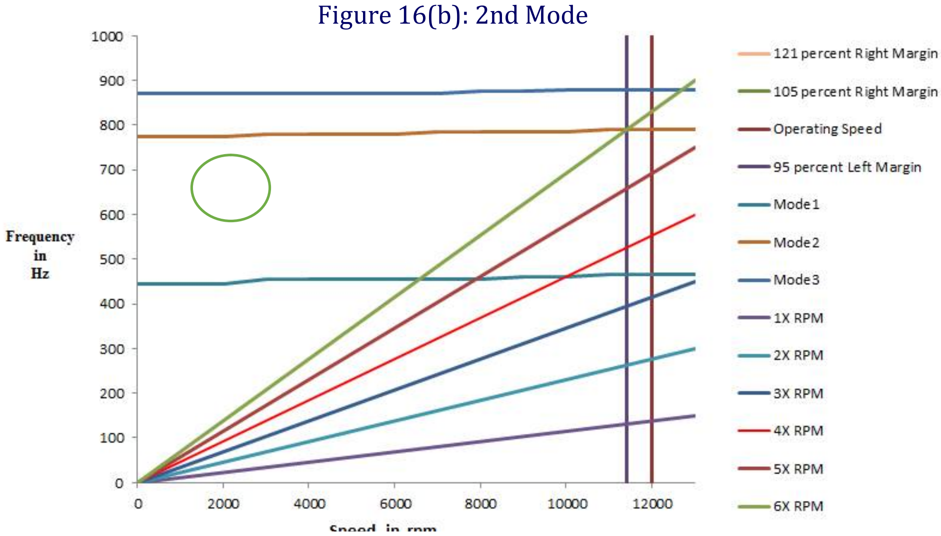



Figure 17: Campbell diagram of optimized model

Figures 16, it clearly represents that all of the first 6 gas engine powerful excitations are safely passing with enough bandwidth without any resonance for the considered operating speed, separation margins and 121 % over-speed condition [6].The highlighted area in figure 17 is called as the Exciter Box justifies for the similarities in vibration characteristics. The Exciter box is the area of prime importance which shows the frequency separation margins and mode excitations passing through it.



## XI RUN OUT ANALYSIS

**Case 1: Run out Analysis, when 3, 6,9,12 clock position bolts failed in optimized model.**

The necessity of run out analysis is to find the structural integrity of rotor coupling when the bolts failed in 3, 6,9,12 o clock locations.

For test cases considered for the run out analysis is as follows:

1. Flange arm is unsymmetrical to each other and bolt preload is applied internally
2. Flange arm is symmetrical to each other at hub region of disc and bolt preload is applied externally.
3. Flange arm is unsymmetrical to each other and bolt preload is applied externally.

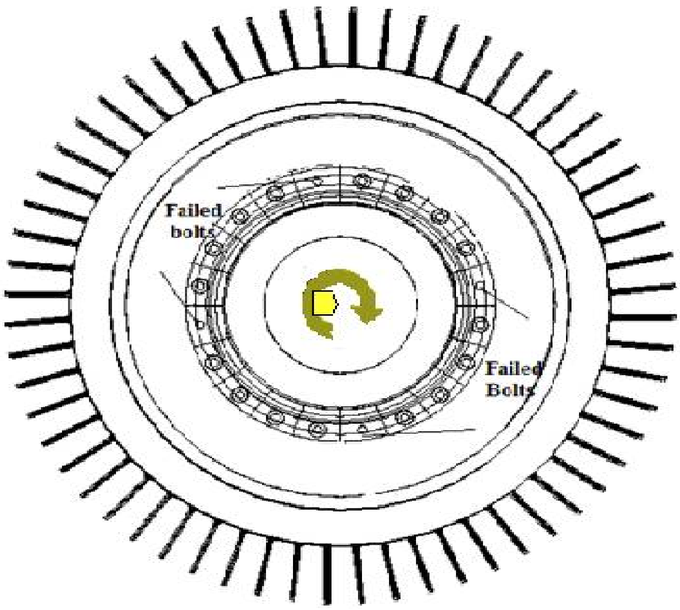



Figure 18: 3, 6,9,12 clock position bolts failed

- Stresses Acting on Adjacent Blades:

When the bolt fails in a rotor coupling, the bolts will not bear any load acting on it. Hence the stress acting on adjacent bolts will be higher compared to other bolts [11]. The equivalent stress observed in optimized configuration is 658 MPa.

## XII. CONCLUSION

- Linear design-optimization tool is used to reduce the weight of a turbine disc. The final weight was reduced by 10 per cent of the actual weight of baseline model. Simultaneously, all the constraints were satisfied and a feasible design is obtained.
- The advantage of general purpose linear optimization software is its capability to handle optimization problem with multiple design constraints using linear codes.
- The results obtained from numerical and simulations method for rotor stresses are matching with an error of less than 8%. The hoop stress is more dominating than radial stress.
- Run out analysis is also carried out to check the stress bearing capacity of adjacent bolts when adjacent bolts are failed.
- Campbell diagram of optimized model is plotted to represent frequency separation margins and mode excitations.
- Analysis is carried out using commercial FE package.

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