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ELEMENTARY EXPERIMENTAL STUDIES ON STRESS WAVE PROPAGATION IN BARS

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Abstract: Studies on stress waves in solids and liquids and shock waves in gases are topics to the specialists in our country and do not find a place in our engineering or science undergraduate (or even post graduate) curriculum. This paper narrates stress wave propagations in solids, in general and longitudinal waves in bars in particular. The genesis of stress waves is discussed and manners in which stress waves are set up in bars are described. Theoretical fundamentals for studying stress waves in bars are briefed. Experimental apparatus required to study (sense, monitor, record and process) stress wave propagations and reflections in a bar caused due to impact at one end are described. Some further extensions of these experiments are discussed for the reader to ponder and progress. This is an attempt to create interest among the students of solid mechanics to undertake such studies.
Keywords: stress waves in bars, experimental studies.

INTRODUCTION

Stress is the internal resistance of a body to external forces applied on it. Normally forces are applied gradually over a period of time so that stress and displacement fields in the body are continuous. As such overall (global) equilibrium is established as the load is applied. It the loads are applied over a very short period of time (sudden displacements within the volume of the body as in earthquakes, impact of a projectile or blast due to an explosion over the surface) local stresses are created and the rest of the body will be unaware of these at the moment. These stresses will be conveyed to the rest of the object over a finite period of time in the form of stress waves.

These disturbance waves created locally will be transmitted to remote areas through the body as well as over the surface. Naturally the former is called body waves and the latter surface waves. Body waves (push-pull) are compressive – tensile stress waves and surface waves are shear waves. Further classifications as spherical waves, longitudinal waves (both push-pull) and Rayleigh, Love, Lamb waves (all surface) are defined. A good book to start a study of stress waves in solids is one of Kolsky [1, 9].

In the following some basic theoretical equations governing longitudinal stress waves in bars that can be found in many books are reproduced, as a starting point. Finer aspects such as dispersion of waves are not touched. Experiments that were performed with a minimal cost and some results so obtained are described and discussed. Extensions to the experiments and interpretations are suggested.

THEORETICAL BACKGROUND

One of the first consistent analysis of longitudinal stress wave propagations and dispersions in bars was given by Pocchammer and Chree is described by Love [2].Experimental studies started with the efforts of Hopkinson [3] to assess the energy in explosives. Stresses caused in a bar by an impact of another bar of the same material and diameter as well as solution for stresses at the interface of the face of a bar indented by a sphere using Hertz's contact theory are given in the classic book by Timoshenko and Goodier[4].The equation governing the stresses created in a ball - bar impact, due to Volterra, is illustrated by Graff [5].

Collinear impact of a bar on another bar: The stress pulse induced at the interface due to a bar of diameter D and length L moving at a velocity v, coaxially impacting a stationary long bar of the same diameter and of the same material is shown to be of a rectangular shape with a stress of [6]

$$\sigma = \rho c v \quad \textbf{(1)}$$

and a time period of

$$\tau = \frac{2L}{c} \quad \textbf{(2)}$$

Here $c = \sqrt{\frac{E}{\rho}}$ is the velocity of stress pulse motion in the bar, E and ρ are the modulus of elasticity and density of the bars. This stress pulse causes stresses and accelerations at its current location, both due to imposed particle motions. By considering the local equilibrium, taking into account the inertial forces due to accelerations and stress gradients, the transmission of the pulse is shown [6] to be governed by the one-dimensional wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \textbf{(3)}$$

Here u is the particle displacement.Collinear impact of a sphere on a bar: The stress pulse due to a ball of mass m impacting a bar of radius R coaxially, both bars of the same material (E, υ) is given by [5]

$$m\frac{d^2 u}{dt^2} + \frac{m.c}{EA} K \left(\frac{du}{dt}\right)^{3/2} + K u^{3/2} = 0 \quad \textbf{(4)}$$

where u is the combined compression of the ball, and

$$K = \frac{2}{3}\frac{E.\sqrt{R}}{(1-\upsilon^2)} \quad \textbf{(5)}$$

can be termed as a combined stiffness of the bar and the ball. The expression for stresses varying with time as dictated by the magnitude of compression is

$$\sigma = -\frac{K u^{3/2}}{A} \quad \textbf{(6)}$$

The shape of the stress pulse obtained from this analysis is shown in Figure.1 and will be discussed later.

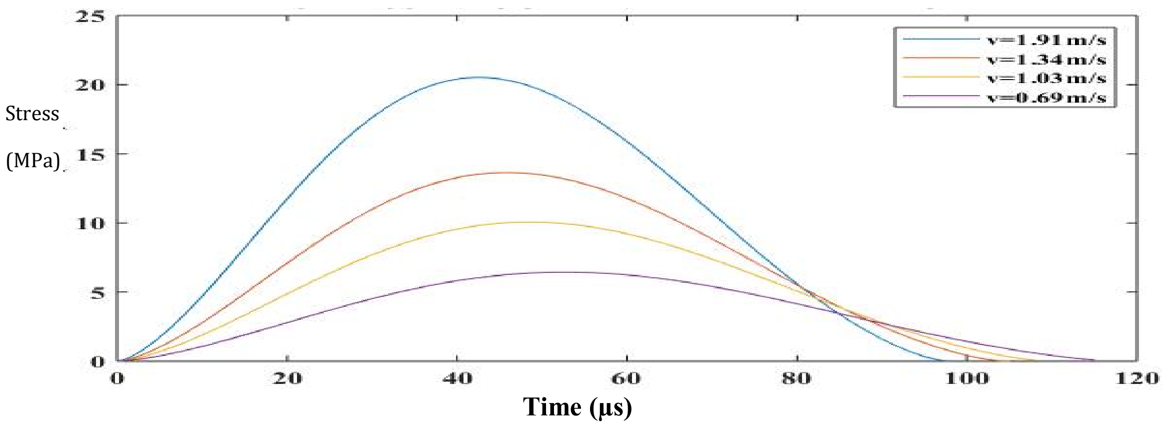

Figure. 1 Stress pulses during ball bar impact

If the bar impacted is of finite length, L, free at the distal end, the impact induced compressive stress pulse gets reflected as a tensile stress pulse of the same detail. In bars of finite length, the stress pulse travels up to the distal end, where the bar will be terminated by a rigid stopper (fixed, no particle displacement possible) or has no constraints (free, no stresses exist). In the former case a compressive stress wave is reflected as an identical compressive tensile pulse and in the latter a compressive stress pulse is reflected as an identical tensile pulse (and vice versa in either case. Effect of discontinuity of area of the bar: Consider a long bar with a step change of cross section impacted at the larger end. Let the areas of cross section of the impacted and distal parts A_1 and A_2 and for generality the densities and velocities of wave propagations be ρ_1& $\rho_2$and c_1&c_2 respectively. At the step a part of the stress pulse goes through to the smaller section and a part of it gets reflected back. By considering equilibrium of forces and continuity of displacements at the section of discontinuity the relationship between σ_I, $\sigma_T$andσ_R is the incident, transmitted and reflected pulses can be shown, see Johnson [6]

$$\sigma_T = \sigma_I - \sigma_R \quad \textbf{(7)}$$

$$\frac{\sigma_T}{\sigma_I} = \frac{2A_1\rho_2 c_2}{A_1\rho_1 c_1 + A_2\rho_2 c_2} \quad \textbf{(8)}$$

And

$$\frac{\sigma_R}{\sigma_I} = \frac{A_2\rho_2c_2 - A_1\rho_1c_1}{A_2\rho_2c_2 + A_1\rho_1c_1} \quad \textbf{(9)}$$

If the mechanical impedances ρ_1c_1 and ρ_2c_2 are equal then equations **(8)** and **(9)** can be reduced to

$$\frac{\sigma_T}{\sigma_I} = \frac{2A_1}{A_1+A_2} \qquad \frac{\sigma_R}{\sigma_I} = \frac{A_2-A_1}{A_2+A_1} \quad \textbf{8(a), 9(a)}$$

The effect of the relative impedances will be interpreted in the following sections. In the following, the experiments carried out are detailed and discussed.

EXPERIMENTS

In the experiments described in the following, a 30mm steel ball bearing was used as the impactor and stainless-steel bars with suitable sensors were used. Firstly, a series of tests were conducted on a straight bar of 20mm diameter bar of 1.2m length. In the second case a bar initially of 25mm diameter was turned down to 18mm over half of its length. Impacts were effected at either end. A simple, but complete, experimental set up is shown in Figure.2. The setup has wooden plank base with a number of vertical posts that helped to suspend the strain gauged test bar and the striker. The strain gauge amplifier that monitored the strain gauges and the oscilloscope, with a typical trace are shown.

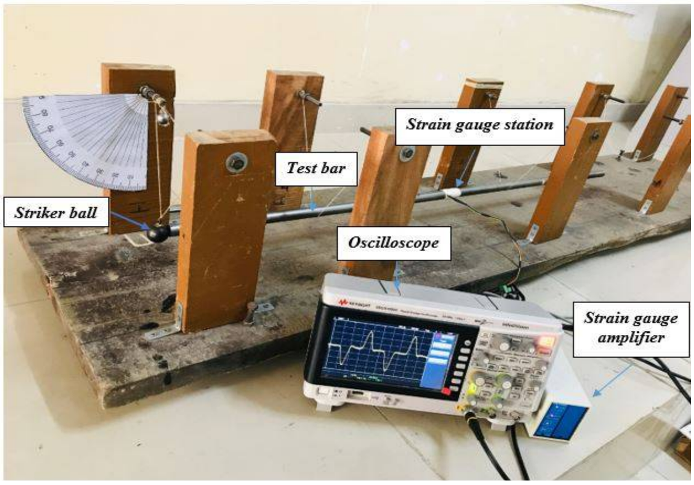

Figure. 2 Labelled photograph of a set up to study stress wave propagations in bars

To strike the bar with the ball, the latter that is hung by threads tied pivoted to the first two posts on the base was pulled, holding the threads taught, and released. The angle of release noted on the calibrated plate, seen in the figure, and the height of the pivot from the centre of the ball provides the velocity of impact.

Metal foil strain gauges were used as sensors to detect stress pulses in the bars. (These are electrical resistance sensors that are pasted to the base material and experience the expansion /contraction of the base material. This change in dimension causes a change in resistance and hence the voltage across its ends.) Instrument grade gauges (6mm gauge length, 3mm width, 350Ω gauge resistance and gauge factor of 2, from Vishay International, USA) were used on the stepped bar at two sections, quarter way from the ends, mounted professionally by M/s SYSCON, Bangalore. Ordinary strain gauges (5mm gauge length, 5mm width, 350Ω gauge resistance and gauge factor of 2 bought from Amazon) were mounted in house on the 20mm diameter straight bar at its central section. In all cases, four strain gauges were used at a section at 90^0 to each other. Two gauges were in the longitudinal and two were in the circumferential direction at each section. The axial gauges were at 90^0 to the lateral gauges.

Strain gauges and adhesive used should be compatible with the base material. Mounting strain gauges is a specialist job. It can be learnt and practiced by any interested person. Standard procedures recommended for preparing, locating, marking and cleaning the surface, handling the gauges (using tweezers) laying them on a cello tape and such have to be followed faithfully. A good description of strain gauge mounting procedure can be found, see for example [8]. After all, four gauges were mounted at a section (took a few hours) the gauges were connected to form a Wheatstone bridge. A lateral gauge was adjacent to an axial gauge, thus the axial and lateral gauges were in opposite arms. Four lead single strand wires were connected to each of the terminals of the bridge. These four lead wires were connected to a strain gauge amplifier the purpose of which is to power (supply a voltage to) the bridge, balance the bridge and receive its output (which will be in millivolts or less) and amplify the same so as to be measured accurately. Two opposite terminals of the bridge have to be connected to the supply and the other two to the amplifying end.

The strain gauge amplifier is a specialist instrument that can be used with quarter, half or full bridge gauge configurations and the procedure recommended by the manufacturer for connecting the strain gauge leads must be followed. A simple strain gauge amplifier can be put together by an interested enthusiast and one such was used in some experiments reported. The only constraint on the amplifier is that it should have a high frequency range (bandwidth), dc to 50kHz (minimum), and it should have a gain of more than 500. The voltage output from a balanced bridge as described above can be shown to be [7]

$$\Delta V = \frac{G*GF*2(1+\upsilon)\varepsilon}{4}.V \quad \mathbf{(10)}$$

where V is the excitation voltage supplied to the bridge, ε is the longitudinal strain in the bar, υ its Poisson's ratio, G is the gain, GF is the gauge factor (defined as the ratio of resistance strain to mechanical strain) and υ is the Poisson's ratio of the bar material. The output from strain gauge bridge, suitably amplified by the SGA was fed to a digital storage oscilloscope. Eq (10) can be interpreted to find the stress in a bar with an elastic modulus of E as

$$\sigma = E\epsilon = 4\frac{\Delta V}{V}\frac{E}{G*GF*2(1+\upsilon)} \quad \mathbf{(11)}$$

When the ball impacted the bar, the strain signal induced due to stress created travelled through the sensor station. The voltage signal from the sensors suitably monitored by the SGA was sent to and stored in the oscilloscope. This was transferred to a computer for further processing.

RESULTS

Straight bars: A typical output signal of the sensors monitored by the amplifier and displayed on the oscilloscope and processed is shown in Figure. 3. The impact velocity judged by the drop height was 1.91m/s in this experiment. The oscilloscope was triggered by the pulse with negative slope at a voltage level of 50mV in this experiment. The compressive pulse created by impact passes through the sensor (central) station at point A. The pulse duration (i.e. contact time between the ball and the bar) is AC (80μs, approx.) and it has a magnitude of about 0.6V.

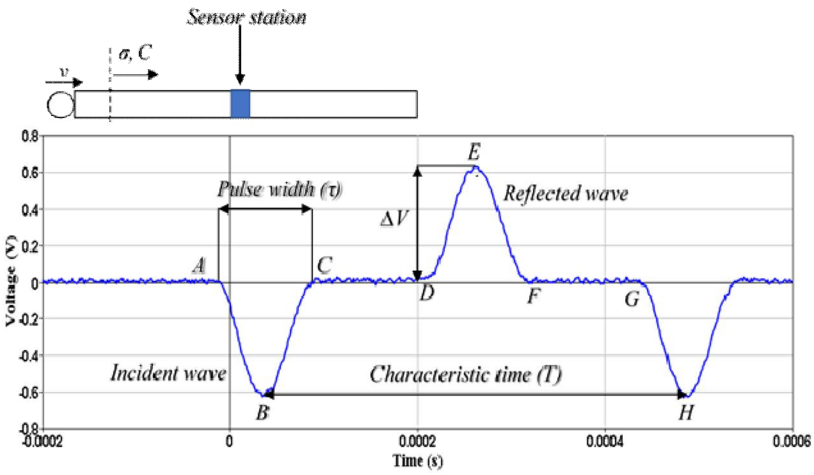

Figure. 3 Voltage vs Time plot for ball-bar impact

The pulse travels to the far end gets reflected at the free end as a tensile pulse, travels back to arrive at the centre at the point E. Further reflection from the impact end as a push wave is sensed at G. The time between the two peaks, BH, is thus the time taken for two complete traverses of the stress pulse

$$T = \frac{2L}{c} \quad \mathbf{(12)}$$

The stress pulses obtained due to impacts at different velocities are shown in Figure. 4. The amplitudes are seen to reduce with the velocity but the variation of contact times (pulse width) is not noticeable. This will be discussed later.

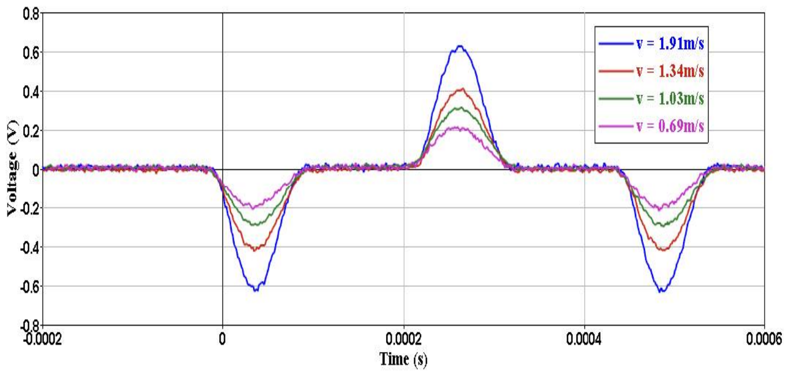

Figure. 4Stress pulses caused in a bar at different velocities of impact

Stepped bars: An SS bar of 25mm diameter was turned down to 18mm (to get an area ratio of 2) over its second half length. Strain gauge sensors were mounted as in the straight bar at the centres of the two half lengths. Impacts were delivered at the bigger end as well as at the smaller end in separate series of experiments. Typical traces of the stress waves propagating in the stepped bar struck co-axially by the sphere at the big end are shown in Figure. 5(a). Part of the compressive incident pulse is reflected at the step as a tensile pulse and travels back while the transmitted part goes ahead to the smaller part of the bar as a compressive pulse. It can be seen that when these pulses are reflected from the respective free ends, the sense of the pulses change again.

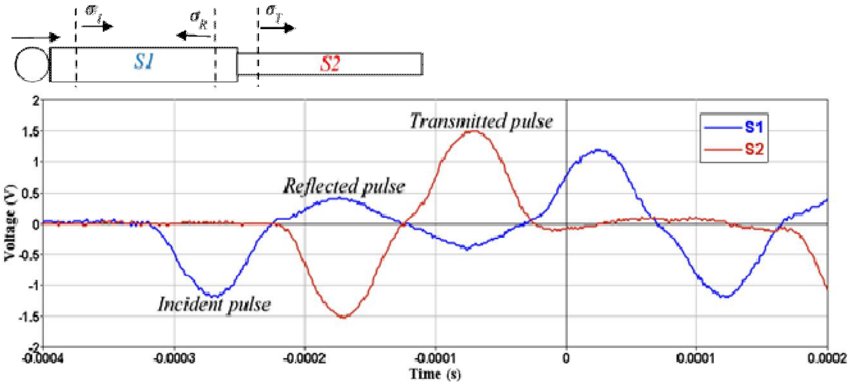

Figure. 5(a) Voltage vs Time plot for stepped bar (impact from bigger end)

The stress pulses in the bar struck at the small end, shown in Figure. 5(b), exhibit similar transmission-reflection and further reflection characteristics. However, the reflection of the incident compressive stress pulse from the smaller section bar is seen to be reflected as a compressive pulse from discontinuity.

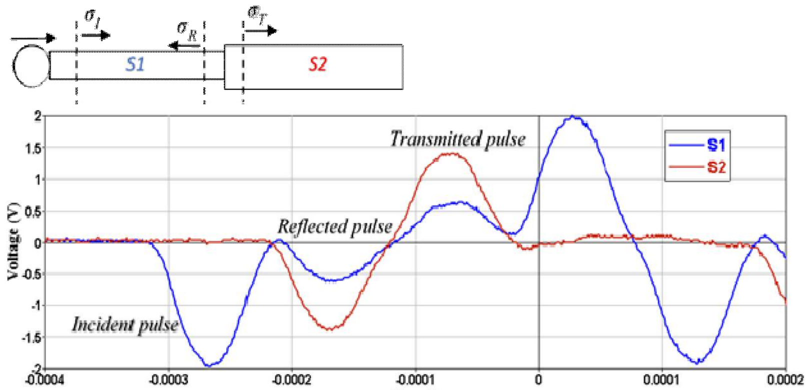

Figure. 5(b)Voltage vs Time plot for stepped bar (impact from smaller end)

Table 1. Experimental and theoretical results of ball-bar impact under different velocities

Release angle θ (degree)	Ball velocity v (m/s)	Characteristic Time ($2L/c$) Experimental T_{exp} (μs)	Pulse width τ_{exp} (μs)	Pulse width τ_{th} (μs)	Output voltage ΔV (V)	Stress (Expt.) σ_{exp} (MN/m^2)	Stress (Theory) σ_{th} (MN/m^2)
90	1.91	456	98.53	98	0.631	13.85	20.5
60	1.34	460	103.98	105	0.425	9.34	13.6
45	1.03	452	111.56	108	0.289	6.35	10.1
30	0.69	452	120.67	115	0.209	4.59	6.4

Table 2. Experimental and theoretical results for stepped bars

Stress ratio	Impact at big end		Impact at small end	
	Expt.	**Theory**	**Expt.**	**Theory**
σ_T/σ_I	1.36	1.30	0.7	0.68
σ_R/σ_I	-.036	-.031	0.3	0.31

The relative amplitudes of incident, reflected and transmitted stress pulses will be discussed in the next section. The reader is left to analyse why the S2 pulse becomes flat after time t=0 in both cases.

DISCUSSIONS

Bar to bar impact tests were not carried out because the alignment arrangements get very elaborate for this simple set up. For a ball to bar impact, the experimental results in Figure. 4 are summarized in Table. 1 along with the theoretical results shown in Figure. 1. These results show a contact time of about 100μs for the 1.9m/s impact. The contact time appears to increase slightly with decrease in velocity of impact, about 120μs for the 0.7m/s impact. However, the voltage amplitudes vary noticeably with velocity of impact .Stresses calculated using equation **(6)**, last column, are higher than experimentally derived stresses. Here, theoretical results are higher than their experimental counterparts. The comparative differences are higher at higher velocities of impact. These aspects need further investigation. However, the ratios of stresses are comparable in both the cases. The ratios of the amplitudes of the voltages of the transmitted and reflected pulses to that of the incident wave pulse, from Figure. 5(a), 5(b) are summarized in on Table. 2. The calculations from theoretical equations, **8(a)** and **9(a)** are also shown alongside the experimental observations These results are agreeable.

CONCLUSIONS

Stress wave propagations due to an axial impact of a sphere on straight and stepped bars are described. Theoretical back ground is briefly explained. Experimental facilities needed are detailed. The experimentally measured quantities are seen to compare well with those provided by theory. It is shown that such studies and their extensions, that form a basis of understanding the behaviour of structures under impulsive and impact loading, can be taken up in any engineering institution imparting undergraduate education in mechanical engineering.

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