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Analysis of MHD Flow of Blood through a Multiple Stenosed Artery in the presence of Slip Velocity

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Abstract— In the present investigation, a mathematical model is developed to study the flow of blood in a multiple stenosed artery employing velocity slip condition under the externally applied transverse magnetic field. Blood is modeled as Herschel-Bulkley fluid to represent the non-Newtonian character of blood in small blood vessels. The expressions for wall shear stress, volumetric flow rate, axial velocity, and core velocity have been derived analytically and graphically. These expressions reveal considerable alterations in flow characteristics due to slip velocity and stenosis. The magnetic field perpendicular to the flow of blood is incorporated which significantly controls the flow patterns. The study provides an insight into the effects of magnetic field and slip velocity on flow rate of blood, wall shear stress, axial and core velocities of blood.

Keywords— MHD flow, slip velocity, multiple stenoses, volumetric flow rate, wall shear stress

I. INTRODUCTION

Cardiovascular diseases, mainly, atherosclerosis (medical called stenosis) is one of the major causes of deaths in the world. These diseases are closely related to the nature of blood flow and the dynamic behaviour of blood vessel. Now a days magnetic therapy is widely employed for treatment of these cardiovascular diseases. The blood is considered as magnetohydrodynamics (MHD) fluid which will help in controlling blood pressure and has probable curative use in the diseases of heart and blood vessel. In case of necrosis, when blood flow to a tissue is reduced or obstructed, local exposure of a magnetic field could potentially result in maintaining blood flow and relaxation of blood vessel. Magnetic therapy may also be helpful for the reperfusion of ischemic tissue or during sepsis. Applying appropriate magnetic field can be effective to the conditions like headaches, travel sickness, poor circulation, muscles sprains, strains and joints pain.

As per the published literature, the idea of applying electromagnetic field in Bio-mathematical research was first given by Kolin [1]. Later, Barnothy [2] reported that the biological systems are affected by the application of an external magnetic field. Haldar et al [3] studied the effect of an externally applied homogeneous magnetic field on the flow characteristics of blood through a single constricted blood vessel in the presence of erythrocytes. Magnetic effect on pulsatile flow in a constricted axisymmetric tube is discussed by Amos el al [4]. A mathematical model for pulsatile flow of blood through a stenosed porous medium with periodic body acceleration under the influence of a uniform transverse magnetic field has been developed by Das et al [5], considering the blood to be a Newtonian and incompressible fluid. Since blood is a suspension of red cells which contains hemoglobin, having iron oxide in composition, it is quite obvious that blood can be assumed as electrically conducting fluid which exhibits the characteristics of magnetohydrodynamics (MHD) flow. If a magnetic field is applied to a moving and electrically conducting fluid, electric as well as magnetic fields will be induced. When these fields interact with each other, a body force known as Lorentz force is produced, which slows down the motion of fluid. Such analysis may be useful for pumping of blood and magnetic resonance imaging (MRI). During surgical procedures, flow of blood can be controlled using magnetic field. In the environment of a uniform transverse magnetic field, treating blood as electrically conducting fluid, Mustapha et al [6] analyzed the flow of blood through irregular shaped multiple stenosed arteries. A mathematical model of bio-magnetic fluid dynamics (BFD), suitable for the description of the Newtonian blood flow under the action of an applied magnetic field, is proposed by Tzirtzilakis [7]. The model is consistent with the principles of ferrohydrodynamics and magnetohydrodynamics and takes into account both magnetization and electrical conductivity of blood. Shit et al [8] explored the effect of externally imposed body acceleration and magnetic field on peristaltic flow of blood through a stenosed arterial segment. Das et al [9] studied the outcomes of a uniform transverse magnetic field on pulsatile flow of blood containing particles through a rough thin walled elastic tube. Varshney et al [10] proposed a mathematical model for the non-Newtonian flow of blood in overlapping stenosed artery in the presence of transverse magnetic field. Mishra et al [11] discussed stenosis of bell shaped geometry to investigate the various flow characteristics of blood through an arterial segment in a pathological state. Utilizing the Herschel-Bulkley fluid model, Jain et al [12] examined the effect of mild stenosis on blood flow, in an irregular axisymmetric artery with oscillating pressure gradient. A mathematical model for magnetohydrodynamics (MHD) blood flow in a stenosed artery under porous medium is developed by Jain et al [13], considering the cosine shaped geometry of the stenosis. Rathod et al [14] studied the pulsatile flow of blood through a porous medium under the influence of periodic body acceleration in presence of magnetic field by considering blood as a couple stress, incompressible, electrical conducting fluid. Singh et al [15] formulated a mathematical model to study the effects of shape parameter and stenosis length on the resistance to flow and wall shear stress under stenotic conditions by considering, laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood through axially symmetric but radially non-symmetric stenosed artery.

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Assuming blood as non-Newtonian fluid (Casson fluid) and artery as circular tube, Bali et al [16] investigated the response of external applied magnetic field on the flow of blood through a multiple stenosed artery. Singh et al [16] examined the effect of magnetic field on flow characteristics through an axially non-symmetric but radially symmetric stenosed arterial segment.

In all of the above studies, none has considered velocity slip at the constricted wall. In view of the possible presence of a red cell slip at the wall, various investigators [18-20] studying blood flow have suggested the likely presence of velocity slip (a velocity discontinuity) at the flow boundaries (or in their immediate neighbourhood). The main objective of the present work is to study the effect of transverse magnetic field along with velocity slip at the arterial wall having a radially non-symmetric multiple stenoses, characterizing blood as Herschel-Bulkley fluid by properly accounting for yield stress of blood in small blood vessels.

II. THE PROBLEM AND ITS SOLUTION

Let us consider an arterial segment having multiple stenoses which are axially symmetric but non-symmetrical with respect to radial co-ordinates. The mathematical expression for the radius of the artery may be written as

$$\frac{\bar{R}(\bar{z})}{R_0} = 1 - A\{(\bar{l}_0)^{s-1}[\bar{z} - m\bar{d} - (m-1)\bar{l}_0] - [\bar{z} - m\bar{d} - (m-1)\bar{l}_0]^s\}; \quad m(\bar{d}+\bar{l}_0) - \bar{l}_0 \le \bar{z} \le m(\bar{d}+\bar{l}_0)$$

$$= 1; \quad otherwise \tag{1}$$

where R_0 is the radius of the artery outside the stenotic region, $\bar{R}(\bar{z})$ is the radius of the stenosed portion of the arterial segment with $\bar{z}$ measured along the axis of the artery, $\bar{d}$ indicates the location of the stenosis, $\bar{l}_0$ is its length, m represents the number of stenosis in the artery, s denotes a parameter determining the shape of stenosis and is referred to as stenosis shape parameter($s \ge 2$). When $s = 2$, stenosis becomes radially symmetric, and a parameter A is given by

$$A = \frac{\delta}{R_0 \bar{l}_0^{\,s}} \frac{s^{s/(s-1)}}{s-1}$$

where δ denotes the maximum height of stenosis at $\bar{z} = m\bar{d} + (m-1)\bar{l}_0 + \bar{l}_0 / s^{1/(s-1)}$.

The equation of motion for flow of blood is given by (Singh et al [17])

$$-\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r}\bar{\tau}) + \mu_0 M \frac{\partial \bar{H}}{\partial \bar{z}} = 0 \tag{2}$$

where $\bar{H}$ is magnetic field intensity, $(\partial \bar{H} / \partial \bar{z})$ magnetic field gradient, $(-\partial \bar{p} / \partial \bar{z})$ is called pressure gradient as $\bar{p}$ stands for pressure at any point $(\bar{z}, \bar{r})$ with $\bar{r}$ measured along radius of the artery and $\bar{z}$ is the axial coordinate, μ_0 denotes magnetic permeability, M magnetization, $\bar{\tau}$ shearing stress.

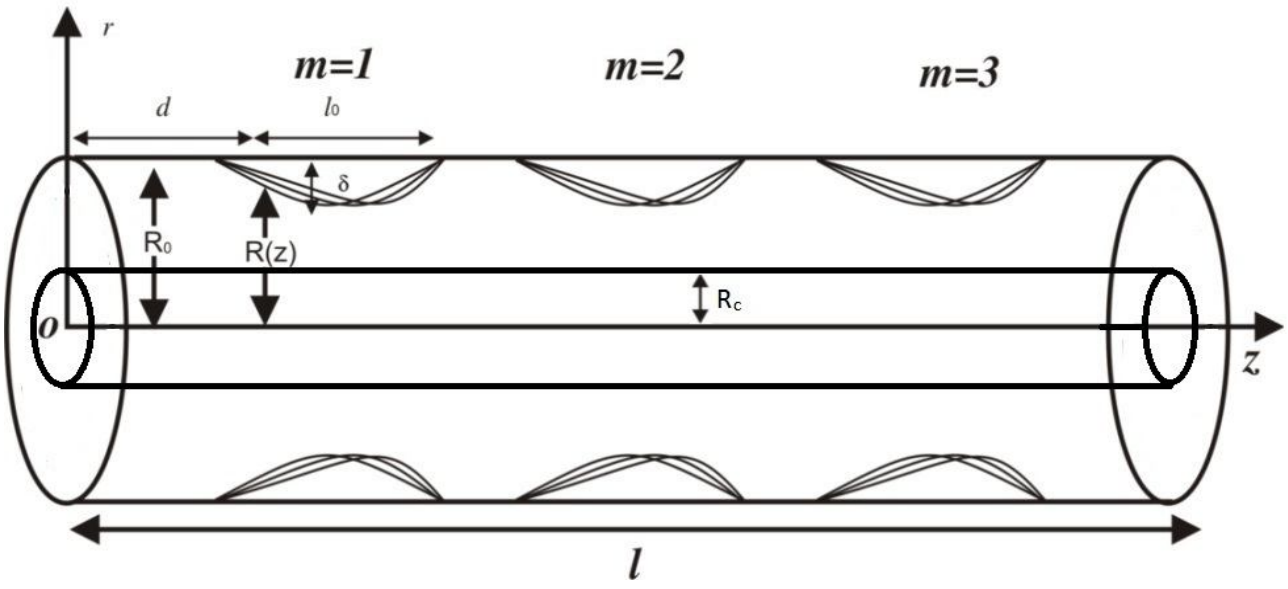

Fig. 1. Geometry of axially symmetric but radially non-symmetric multiple stenosis

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The constitutive equation in one dimensional form for Herschel-Bulkley fluid is expressed as

$$-\frac{\partial \bar{u}}{\partial \bar{r}} = \frac{(\bar{\tau}-\bar{\tau}_0)^n}{k}; \quad \bar{\tau} \geq \bar{\tau}_0$$

$$= 0; \qquad \bar{\tau} < \bar{\tau}_0 \tag{3}$$

where $\bar{u}$ stands for axial velocity of blood, $\bar{\tau}_0$ the yield stress, n the flow behaviour index of blood and k the viscosity coefficient of blood.

Equation (2) and (3) are to be solved subject to the boundary conditions

$$\bar{u} = \bar{u}_s \quad \text{at} \quad \bar{r} = \bar{R}(\bar{z}) \quad \text{(velocity slip condition)} \tag{4}$$

$$\bar{u} = \bar{u}_c \qquad \text{at} \quad \bar{r} = \bar{R}_c \tag{5}$$

where $\bar{R}_c$ is the radius of the core region and $\bar{u}_c$ is the velocity in the core region.

$$\bar{\tau} \text{ is finite at } \bar{r} = 0 \text{ (regularity condition)} \tag{6}$$

The following non-dimensional variables are now introduced as:

$$d = \frac{\bar{d}}{l},\ l_0 = \frac{\bar{l}_0}{l},\ z = \frac{\bar{z}}{l},\ r = \frac{\bar{r}}{R_0},\ R = \frac{\bar{R}}{R_0},\ R_c = \frac{\bar{R}_c}{R_0},$$

$$u = \frac{\bar{u}}{u_0},\ u_s = \frac{\bar{u}_s}{u_0},\ u_c = \frac{\bar{u}_c}{u_0},\ p = \frac{\bar{p}}{\rho u_0^2},\ \tau = \frac{\bar{\tau}}{\rho u_0^2},\ H = \frac{\bar{H}}{H_0}$$

where H_0 is the external transverse uniform constant magnetic field.

Equations (1) – (5) reduce to the following non-dimensional form:

1. The geometry of the stenosis:

$$R(z) = 1 - B\{(l_0)^{s-1}[z - md - (m-1)l_0] - [z - md - (m-1)l_0]^s\}; \quad m(d+l_0) - l_0 \leq z \leq m(d+l_0)$$

$$= 1; \quad \textit{otherwise} \tag{7}$$

2. Equation of motion:

$$-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) + A_1\frac{\partial H}{\partial z} = 0 \tag{8}$$

3. Constitutive equation of Herschel-Bulkley fluid:

$$-\frac{\partial u}{\partial r} = \frac{(\tau-\tau_0)^n}{A_2}; \quad \tau \geq \tau_0$$

$$= 0; \qquad \tau < \tau_0 \tag{9}$$

4. Boundary conditions:

$$u = u_s \text{ at } r = R(z) \tag{10}$$

$$u = u_c \text{ at } r = R_c \tag{11}$$

$$\tau \text{ is finite} \quad \text{at} \quad r = 0 \tag{12}$$

where $B = \frac{\delta}{R_0 l_0^s}\frac{s^{s/(s-1)}}{s-1}$, $A_1 = \frac{\mu_0 M H_0}{\rho u_0^2}$, $A_2 = \frac{\mu}{\rho^n u_0^{2n-1} R_0}$

Integrating (8) and (9) and using boundary condition (10) – (12), the expressions for axial velocity u and core velocity u_c are obtained as

$$u = u_s + \frac{1}{2^n(n+1)A_2}\left(\frac{\partial p}{\partial z} - A_1\frac{\partial H}{\partial z}\right)^n\left[(R-R_c)^{n+1} - (r-R_c)^{n+1}\right] \tag{13}$$

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$$u_c = u_s + \frac{1}{2^n (n+1) A_2}\left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)^n (R - R_c)^{n+1} \tag{14}$$

The volumetric flow rate, Q is formulated as

$$Q = \int_0^R 2\pi r u(r) dr$$

which may be put as

$$Q = \int_0^{R_c} 2\pi r u_c dr + \int_{R_c}^R 2\pi r u dr \tag{15}$$

Integrating (15) and using (13) and (14), the expression for the volumetric flow rate Q may be given as

$$Q = \pi R^2 u_s + \alpha C^n R^{n+3}\left[1 + \frac{2}{n+2}\beta + \frac{2}{(n+2)(n+3)}\beta^2\right](1-\beta)^{n+1} \tag{16}$$

where $C = \left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)$, $\alpha = \frac{\pi}{2^n (n+1) A_2}$, $\beta = \frac{R_c}{R} = \frac{\tau_0}{\tau_R}$

When τ_0 / τ_R □ 1 (16) reduces to

$$Q = \pi R^2 u_s + \alpha C^n R^{n+3}\left(1 - \frac{n+2}{n+3}\beta\right)^n \tag{17}$$

Formulating the wall shear stress as

$$\tau_R = -k\left(\frac{\partial u}{\partial r}\right)_{r=R} \tag{18}$$

On differentiating (13) and using it in (18), the wall shear stress τ_R may be expressed as

$$\tau_R = \frac{k}{2^n A_2}\left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)^n \left[(R - R_c)^n\right] \tag{19}$$

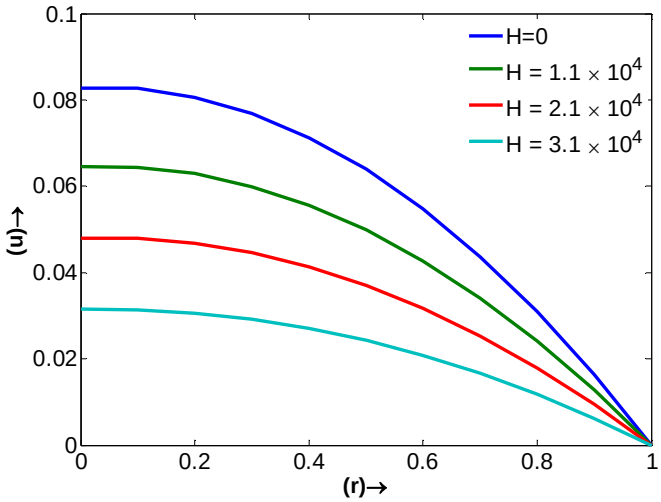

Fig. 2. Variation of axial velocity of blood along radial distance for different values of magnetic field intensity (H)

III. RESULTS AND DISCUSSIONS

All this section, numerical results have been made available to explore the effects of slip velocity, magnetic field, shape parameter on the axial velocity, volumetric flow rate and wall shear stress etc. The present problem with extended ideas considered herein is difficult to handle but computations and graphical representations with MATLAB 7.0 make it easier. Figures 2 and 3 reveal the variation in axial velocity of blood along radial distance, velocity curve shifts towards the origin as radial distance increases. It can be seen through figure 2 that magnetic field controls the velocity of blood remarkably whereas figure 3 suggests that slip velocity at the constricted wall enhances the velocity profile of blood, so,

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it may be concluded from these figures that the response of slip velocity reduces the effect of induced magnetic field intensity ($H = \partial H/\partial z$). The deviation in axial velocity with stenosis height and shape parameter is shown in figure 4. Axial velocity of blood increases with decreasing stenosis height and increasing shape parameter. This proposes that as stenosis loses its symmetry, velocity of blood goes on decreasing. Figure 5 illustrates that like axial velocity, core velocity shows similar variations with radial distance and magnetic field.

Figure 6 depicts that the volumetric flow rate remains constant in non-stenotic region, starts decreasing as blood enter into the stenotic region, it becomes least at the peak value of stenosis height and again starts escalating and reaches to same constant value when blood comes to the normal region. The flow rate shows same movements for the second stenosis. It can be observed that magnetic field can be applied to manage flow rate. Changes in flow rate due to parameter determining the shape of stenosis and slip velocity are shown in the figures 7 and 8. One may see that slip velocity and shape parameter augment the volumetric flow rate while flow rate diminishes with growing stenosis height. Figures 9 and 10 demonstrate the augmentation in the wall shear stress due to increasing stenosis height but the wall shear stress can be taken under control using magnetic field. The shear stress is greater in case of symmetry stenosis that in case of non-symmetric one.

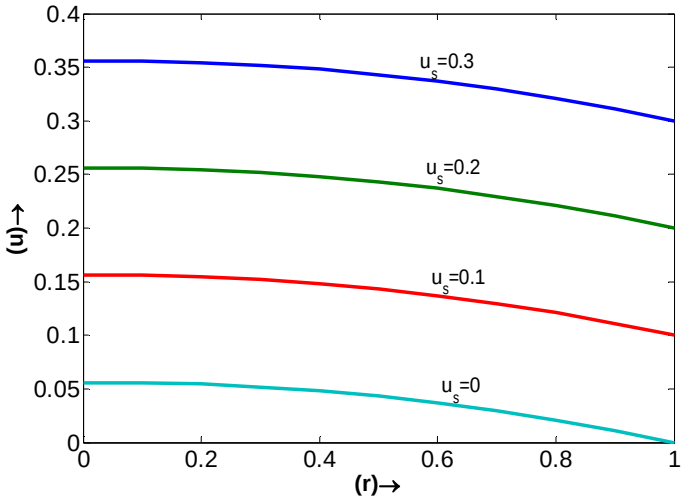

Fig. 3. Variation of axial velocity of blood along radial distance for different values of slip velocity (u_s)

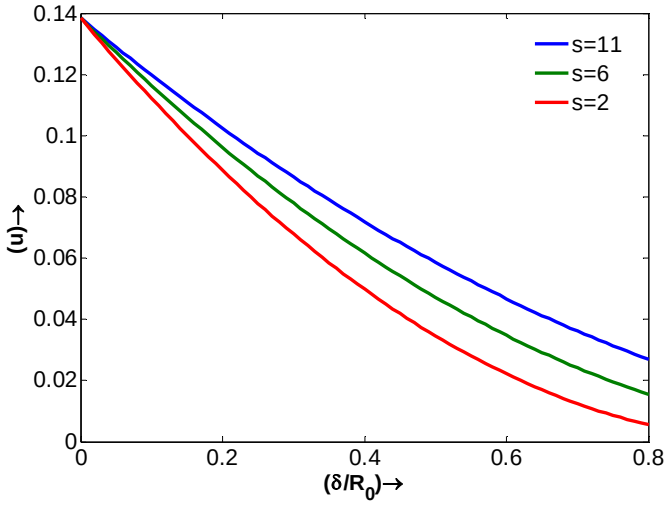

Fig. 4. Variation of axial velocity of blood with stenosis height for different values of shape parameter (*s*)

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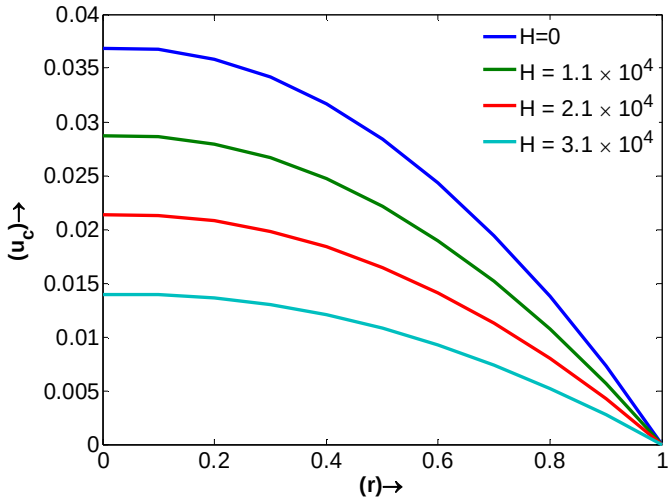

Fig. 5. Variation of core velocity of blood along radial distance for different values of magnetic field intensity (*H*)

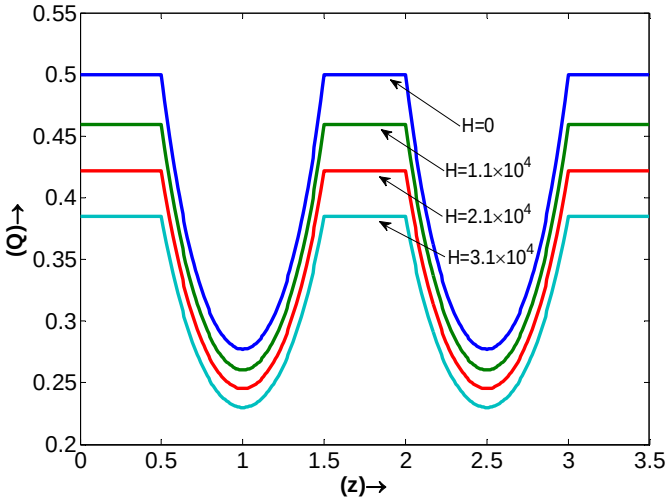

Fig. 6. Variation of volumetric flow rate along axial distance for different values of magnetic field intensity (*H*)

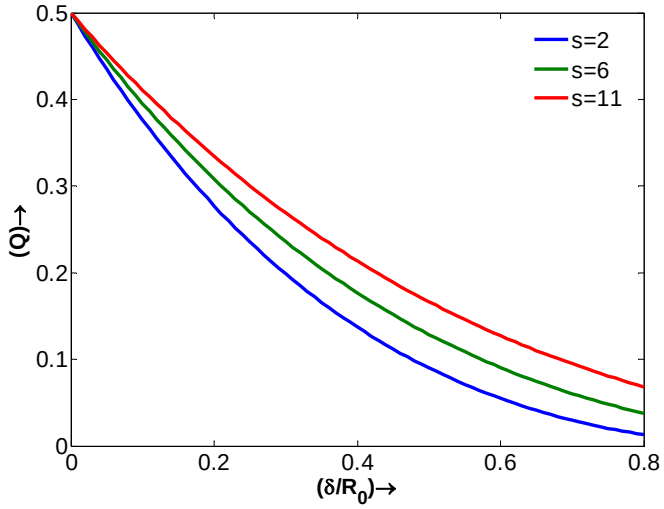

Fig. 7. Variation of volumetric flow rate with stenosis height for different values of shape parameter (*s*)

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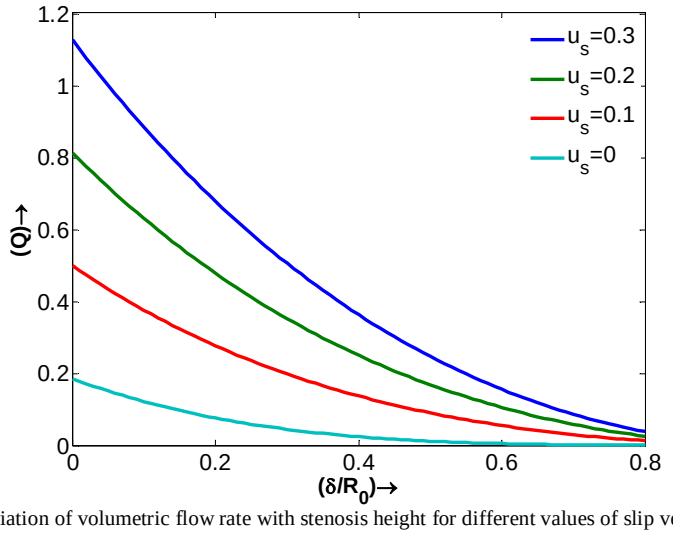

Fig. 8. Variation of volumetric flow rate with stenosis height for different values of slip velocity (u_s)

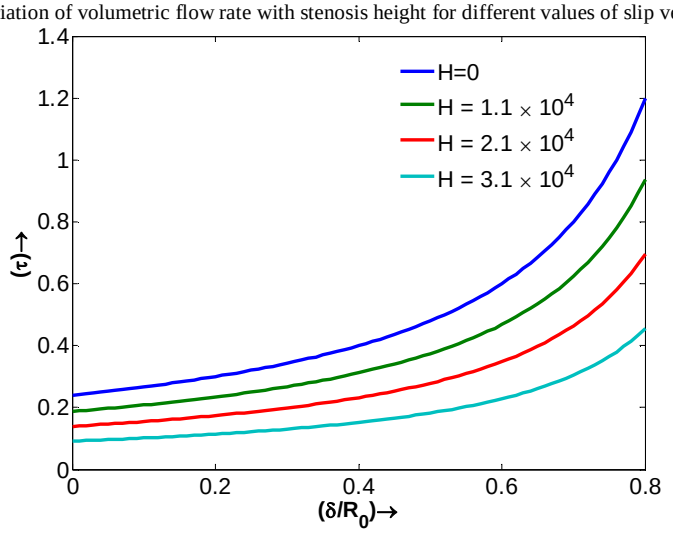

Fig. 9. Variation of wall shear stress with stenosis height for different values of magnetic field intensity (H)

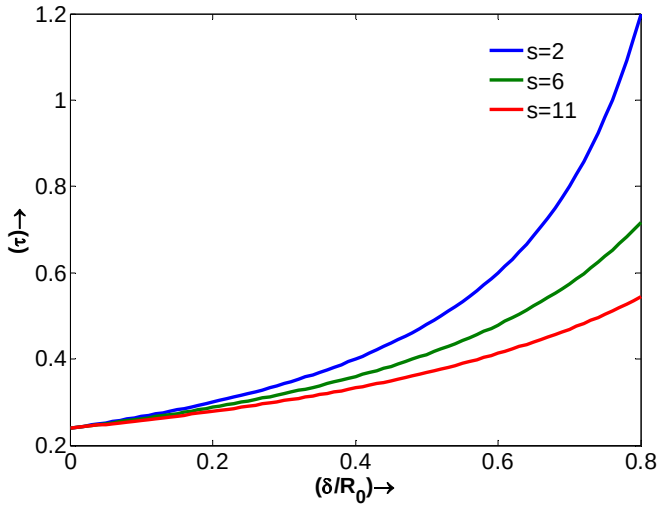

Fig. 10. Variation of wall shear stress with stenosis height for different values of stenosis shape parameter (s)

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IV. CONCLUSIONS

In the present analysis, we have developed a mathematical model to investigate the influence of slip velocity and magnetic field on velocity profile, volumetric flow rate and wall shear stress. Blood is characterized as Herschel-Bulkley fluid model. It is observed that magnetic field reduces the flow characteristics amazingly. Also the height of stenosis significantly affects the velocity, wall shear stress and flow rate. Knowing about slip effect, appropriate magnetic field can be applied to manage and control the flow behaviour of blood. These investigations may be useful for the medical practitioners to treat the hypertension patients through magnetic therapy and to understand the flow of blood under stenotic conditions.

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